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Some Implications of Differentiation During a Seventh Grade Probability Unit

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by

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Dedication

This thesis is dedicated to my parents, Sarah and Daniel Swancott. I would not have made it this far academically, professionally, or personally without them.

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Abstract

Educators inevitably realize that their classrooms are diverse environments with regards to student interests, learning styles, and abilities. Research has shown that differentiating instruction promotes student learning by taking into account student complexities. Moreover, research has shown that teaching probability with activities and experiments can be advantageous. However, research has not been completed to determine if differentiating activities along with instruction can be similarly beneficial. For the researcher's purpose, probability activities and differentiating instruction were explored simultaneously.

The researcher sought to conduct this investigation in the context of a rural New York private school. Varied activity based instruction was utilized to instruct students who were split into two groups. These groups, an upper differentiation group (UD) and a lower differentiation group (LD), were formed based on their prior knowledge determined by a pretest. Throughout the course of the research, these two groups learned probability through activity based instruction and guided worksheets, with the UD group being offered less support and the LD group being more supported by the researcher. Each day, the students were assigned homework with choices. Upon completion of the unit, students were given a posttest. Surveys and interviews were also utilized to determine student perceptions of mathematics prior to and subsequent to the research. All areas of research offered numerous venues to be statistically analyzed revealing the pros and cons of the differentiated instruction.

Chapter One: Introduction

Statement and Significance of the Problem

Novice teachers, and even some teachers with years of experience, struggle with having to teach students of varying ability levels in one classroom. This problem is amplified in secondary education when teachers have, at most, one hour to teach a concept to diverse learners. More specifically, some students grasp the idea the first time the teacher explains it, while others need numerous repetitions and a variety of explanations to understand the idea at hand. Moreover, there are a handful of students in between these two extremes. Probability is especially troublesome because it utilizes numerous math concepts. Students need to draw on their prior knowledge of fractions, decimals, and percents while simultaneously applying these in a new context that builds upon itself as the unit progresses. Because of this, the gap between gifted and struggling students is widened. Although some promise has been shown for activity based instruction in probability, for one teacher to balance all student variance is difficult and frustrating if differentiation is not properly utilized (Gurbuz, 2010).

It is known that differentiation is advantageous to student learning because it takes into account various learning styles (Strong, Thomas, Perini, & Silver, 2004). However, teachers struggle with implementing this teaching practice in a practical manner. Classroom management, content knowledge, and sheer will are only some of the factors that stand in the way of teachers meeting the unique needs of their students (Tomlinson, Callahan, Tomchin, Eiss, Imbeau, & Landrum, 1997). Therefore, even though instruction and assessment should be modified for the variety of learning styles and abilities in the classroom, this teaching practice is not always used as it should be, causing students to struggle and teachers to become exasperated. With this in mind, teaching practices that have been shown effective for probability and proven differentiation techniques should be unified to create what can be assumed to be an effective and motivating learning environment. The researcher created such a learning environment to model this vision and saw outcomes that previous research had not.

Purpose and Research Questions or Hypotheses

The purpose of this study was to discover "Some Implications of Differentiation during a Seventh Grade Probability Unit." Specifically, did the class as a whole perform better than the previous year's class when differentiation of instructional activities was not used? Also, did homework with choice influence individual student achievement? Finally, did the students' perception of mathematics change as a result of the researched instructional methods.

The null hypotheses of this study were that there would be no significant difference between the two year's unit test grades, homework completion and the posttest scores, and the survey scores for perceptions of mathematics received by students who had their classroom instruction and assessment intentionally differentiated. First, the grades from last year's class and this year's class on a control unit test were statistically compared to ensure that there was no significant difference between the groups when taught in a similar manner. To confirm or reject the null hypotheses, statistical analyses were performed on the data. The t-value, which represents the probability that there was no difference, was then analyzed. Comparing it to a predefined t-value from the table at a 0.05 level allowed the researcher to reject or fail to reject the null hypothesis. Moreover, a correlation coefficient was found for unrelated samples and the r-value was compared to -1 and 1 to determine if the researcher would reject or fail to reject the null hypothesis.

Theoretical Perspective

In the statement of the problem, it was acknowledged that there are ambiguities in teachers' instruction. Although most educators know that differentiation should be at the heart of their teaching practice, it is often implemented extraneously. Moreover, studies have shown that activity based instruction is an ideal way to teach probability. It engages the students, makes the content more relevant, and helps them construct their own learning with the aspiration that it will be better recalled and applied in the future. However, there have been no studies examining the effectiveness of differentiating the activities and assessments that are often used as tools to teach probability. Therefore, it was the viewpoint of the researcher that combining these two areas would result in the fruition of the aforementioned hypotheses.

Because of the importance of teaching students in alignment with their individual needs and the complexity of probability, this research study is to be taken professionally. Previous research was used as a footstool to greater understanding and it was ensured that the methods were reliable in order to ensure validity of the interpretations and uses of the results. Thus, the researcher of this study aspired to propose research that was exemplary in idea and rigor so that other researchers would be encouraged to follow suit.

Definition of Terms

This proposed research involved a variety of content and teaching practices whose definitions should be addressed. For the purpose of this study, probability is determining how likely a certain event is. Learner objectives for probability included students understanding and calculating theoretical and experimental probability. They would understand and find sample spaces. Moreover, they would understand compound events and calculate the probabilities of independent and dependent events. Finally, students would use permutations and combinations to find possible arrangements. One of the ways probability can be taught is activity based instruction. Activity based instruction can be defined as students completing "experiments and then … [discussing] their experiments and conclusions with each other" (Gurbuz, 2010, p. 1058). By doing this, students construct their own understanding of the material that can presumably be better recalled and applied.

Unfortunately, as previously addressed, any teaching of probability is futile without differentiating instruction for various learners. Differentiation is when "instruction is [adjusted] to respond to …diversity" (Tomlinson, 2004, p. 519) or, in other words, the teacher responds to learner variance as opposed to the students having to fit into a rigid structure established by the teacher. Learning variance is a social cognitive issue and teachers should be attuned to how their students learn and be flexible to those needs. Teachers can accommodate their diverse learners through varying instruction, as well as assessments.

Delimitations and Limitations of the Study

The proposed study was delimited in that teacher accessibility is accounted for. The researcher, a certified teacher, instructed the class and was proficient in the content area of probability and well read in differentiation. Moreover, the researcher was the mathematics teacher of the participating seventh grade class. Therefore, the researcher spent almost an entire school year with the study participants before the research period. This permitted the researcher to discover the students' individual learning styles and how to manage them as a class. Having this stability was important because the research sought to get at the heart of the implications of differentiation, so there should have been few external pedagogy factors getting in the way.

Nevertheless, the study was limited in a couple of blatant ways. First, it was mainly an exploratory study. The researcher sought to understand the implications of differentiation in one seventh grade class in a rural New York school. This class had eighteen students, which was a small sample size for quantitative research. Nevertheless, the research was taking place in order to gain a firm understanding of the influence of this teaching technique on a specific sample. It was hoped that future seventh grade instruction at this school would be improved as a result. However, the researcher recognized that the conclusions would be limited to the select school where the research took place or schools with extremely strong similarities to it. It was advised that the conclusions be generalized at the discretion of readers and their opinion of the similarities.

The second most significant limitation was that the study only spanned one probability unit. Although probability was an ideal math concept to use intentional differentiation with, it may be possible that the selected unit brought out the strengths of some students more so than others. If research were to occur over the course of a few units, a more balanced understanding of differentiation could be gathered. Regardless, the content taught while using differentiation was another important factor to take into account when considering relating the conclusions of this study to other situations.

Chapter Two: Review of the Literature

Educators inevitably realize that their classrooms are diverse environments with regards to student interests, learning styles, and abilities. Differentiation is assumed to be advantageous for addressing this complexity, but it was important to examine scholarly research to understand when and how it could be used most efficiently. More specifically, the context for which differentiation would be used was analyzed and understood independently. Once this was accomplished, varying instruction could be applied to the particular concept effectively. For the researcher's purpose, probability and differentiating its instruction were explored.

Teaching Probability

Probability is an intricate math concept to teach. To begin with, it is the culmination of numerous other math concepts. In most curriculums, it is taught at the end of the year, utilizing the other concepts learned throughout the year such as fractions, decimals and percents. Moreover, it is taught after numerous other topics such as number facts, algebra, and geometry. Thus, there are numerous reasons that students struggle when being taught probability.

Specifically, there tends to be misconceptions by students when calculating probabilities. A common error is applying the linear or proportional reasoning learned when solving algebra problems to problems relating to probability (Van Dooren, De Bock, Depaepe, Janssens, & Verschaffel, 2003). A study on this misconception revealed that even when students have formal instruction in probability, they still solve some problems using erroneous linear reasoning as opposed to correct probability calculations. More specifically, students in tenth and twelfth grade participating in the research were given the same probability test. There was no significant difference between these two groups even though the students in twelfth grade had taken a probability course while the tenth grade students had not. (Van Dooren, et al., 2003) Therefore, something in the instruction at this particular school in Belgium was allowing students' erroneous thought processes to persist.

Another study showed that misconceptions developed in grade school can persist though college. In this study, college students' ability to take a weighted mean was analyzed. It was found that although students could perform this computation, they were usually unaware of why or when to do so, relying on blind application of a learned skill. (Pollatsek, Lima, & Well, 1981)

These two misconceptions are connected. In both, students know basic math processes but are unaware of why or when to use them. If students cannot accurately employ their computational skills, they will be ill-equipped to answer applied questions. Although this information cannot be generalized to all students' learning of probability, it is fair to argue that misconceptions are prevalent and instruction should be examined.

To alleviate this, numerous studies have been done on how to better teach probability. A common thread in the research is activity based instruction. Activity based instruction can be defined as students completing "experiments and then …

[discussing] their experiments and conclusions with each other" (Gurbuz, 2010, p. 1058). Specifically, for probability, the experiments consist of having students actually roll dice, spin spinners, toss coins, and draw cards. It has been shown that these types of experiments are extremely advantageous for the students and practical to do (Dunn, 2005). For younger students, the standard manipulatives are helpful as students realize what can happen in different situations and their likelihood. As students become more sophisticated in their thinking, it is important to have variety in these manipulatives so that students do not get bored thinking that they already know the probabilities. (Dunn, 2005)

Activity based instruction maximizes student learning with these manipulatives. Instead of the teacher using the manipulatives, the students are allowed to predict outcomes and assess the results by performing experiments on their own. When operating as it should, activity based instruction helps the students develop cognitively by finding mathematical patterns and problem solving when what they previously thought would happen, does not. They are able to see mathematics in action and apply it in ways that make it meaningful and relevant, not just some mundane topic adults and schools think they should know. (Gurbuz, The effect of activity-based instruction on conceptual development of seventh grade students in probability, 2010; Gurbuz, et al., 2010)

While teaching the intricate topic of probability, it is essential that students develop in their cognition of the topic. Manipulatives and activity based instruction have been shown to foster students in developing their own understanding of probability. Furthermore, there are other instructional methods that can encourage students' cognitive development in probability. Similar to activity based instruction, there are didactic methods which utilize a variety of teaching strategies such as technology and problem solving, while most importantly taking into consideration how the students processed their learning (Castro, 1998). This is in opposition to epistemology based instruction which is logically structured so that instruction follows a linear pattern, regardless of the students' cognitive processes. Castro (1998), in comparing these two instructional methods, determined that there was a significant difference in favor of the didactic group in terms of probability calculation and probability reasoning which proves promising for instructing through conceptual change. This was rationalized by reemphasizing the structure of the didactic method. Importantly, student misconceptions were addressed and not ignored to follow a linear path of instruction set forth by the teacher. In addition, the use of hands-on experiments relate to the students and their use of probability. By doing this, students are able to see the relevant need for probability, seek to understand the algorithms behind probability calculations, and are consequently more successful in their calculations than those instructed with epistemology. (Castro, 1998).

Greenes (1995) affirms this research by assessing student investigation. It is confirmed that when students personally explore mathematic concepts, they gain a more thorough and lasting understanding of the material. This is with special emphasis on the basic math concepts of arithmetic, algebra, geometry and probability, and statistics. A theoretical perspective including the constructivist approach and student centered learning revealed by Greenes (1995) endorses the activity based instruction proven beneficial (Gurbuz, The effect of activity-based instruction on conceptual development of seventh grade students in probability, 2010; Gurbuz, et al., 2010) and didactic methods (Castro, 1998) when focusing on the foundational topics of mathematics, such as probability.

Teaching Exceptional Learners

In addition to the difficulties of teaching probability, educators must also overcome the students' varying learning ability levels. A variety of approaches have been made to differentiate instruction. The first of which is tracking, or placing students in different classes based on ability level. Although this differentiation is often criticized because of the potential implications this can have on students' self-esteem or self-concept, research has been performed to test these criticisms (Chiu, et al., 2008; Trautwhen, Ludtke, Marsh, Koller, & Baumert, 2006). One statistical analysis did reveal that students in lower tracks have lower self-beliefs, while students in higher tracks have higher self-beliefs (Chiu, et al., 2008). This confirmed some of the previous research done on tracking but contradicted the advocating research. Taking into account student grades, this correlation diminished leaving the researchers to believe that more important than tracking's influence on self-belief are grades and the perceived teachers' opinions that go along with them. (Chiu, et al., 2008). Similar results were found by Trautwhen, et al. (2006) who affirmed "that differentiated grading practices, and not psychological assimilation effects per se, were the driving force behind the track differences observed in math self-concept" (p. 801). This is another affirmation that grades, not tracking itself, influence students' beliefs in their potential. Research on this is important because students are acute to their peers and it can be assumed that any form of differentiation would be noticed and evaluated by them. Therefore, even differentiation within the classroom has the potential to harm students' self-esteem and self-concept. Fortunately, the research shows that students are not guaranteed to be harmed by being separated based on ability levels. However, teachers should still implement this practice with caution and grace, especially when it comes to grading varying students.

Tracking is not the only way in which schools can differentiate. A great deal of differentiation should occur in the classroom with the teacher facilitating or as Tomlinson, et al. (1997) defines it, becoming an architect of communities of learning. Their research focused mainly on what strategies experienced teachers utilized, as opposed to what strategies novice teachers lacked when trying to address the academic diversity in their classroom. It appeared as though preservice teachers were aware and willing to differentiate and recognized that developing their ability to do so will make them more effective teachers. Novice teachers were also willing to problem solve to figure out more efficient ways to differentiate and seek advice from experienced educators. Unfortunately, beliefs do not always meet practice and the novice teachers often reverted to teacher centered education that was one size fits all. The reason is novice teachers struggled with learning the system, coverage, equal time for everyone, seeing who got it, kids who exceed the standard, kids who fall short of the standard, management for time and coverage, differentiation in the context of uniformity, and discouragement. (Tomlinson, et al., 1997)

One of the more specific areas this study looked at was seeing if a curriculum coach would help preservice teachers implement their beliefs and desires. Although no significant difference was seen between those teachers who had a curriculum coach and those who did not, this was probably because the preservice teachers had so many people working with them, such as professors, cooperating teachers, and researchers, that the curriculum coaches just added noise to the surplus of advice being thrown at them. A curriculum coach would more likely be advantageous if available throughout a teachers' first few years of teaching. (Tomlinson, et al., 1997)

Similarly, another study examined how teachers can address learner variance (Tomlinson, The mobius effect, 2004). As Tomlinson, et al. (1997) recognized, novice teachers acknowledge and desire to differentiate. To fulfill this ambition, the first step is becoming a teacher who addresses, and does not cover up, learner variance. Specifically, teachers need to be able to identify their gifted students and remedial learners in order to teach them most appropriately. An understanding of this should be relevant in the classroom, as well as in the broader societal setting. Once the learners are more fully understood, teachers should differentiate the material in order to account for the variance.

Ways in which this can be done are by reassessing the nature of the curriculum and the instruction. (Tomlinson, The mobius effect, 2004) This is not always easy for teachers but is something that is valued and should be implemented (Tomlinson, et al., 1997). Particularly, teachers can evaluate the curriculum so that it promotes student economic, cultural, and personal self-beliefs. By connecting the content to the learners, it will be more meaningful than simply drill work. Taking into account student backgrounds, implications are that these must be personal and well thought out on the part of the teacher. Furthermore, instruction should be both at the perceived level of the learners and strived to bring the learners to a higher academic level. That is to say that the remedial learners, although initially should be instructed as such, should eventually be assisted to function at a higher level. Moreover, the gifted students should be challenged to reach an even higher potential. Therefore, in sum, any attempts at differentiation should be carefully thought out in order to assist every student in reaching their fullest potential. If used merely as a means of getting a classroom on the same page, all potential benefits are lost. (Tomlinson, The mobius effect, 2004)

Teaching Exceptional Learners Probability

It is important to understand how exceptional learners can be specifically taught probability. Broadly, it is known that teachers should be flexible in their instruction and not cling to a rigid structure that will not work for all students. Above all, the classroom should be student centered in order for students to achieve all that they can in the best ways that they can. (Tomlinson, et al., 1997; Tomlinson, The mobius effect, 2004) In order to do this, teachers must have a solid understanding of the content (Liu & Thompson, 2007). To make adjustments that meet student needs, teachers need to understand the core of the content, what every learner should walk away with, and also extensions of the content that will reach remedial learners, as well as gifted students. Therefore, it is imperative that teachers fully understand the topics of probability before teaching them. (Liu & Thompson, 2007) This is something that novice teachers struggle with and often limits them in differentiating. It has been claimed that novice teachers, "lack rich or even adequate content understanding in their domains…practice a sort of ransom selection of solutions to problems" (Tomlinson, et al., 1997). To rectify this, teachers should acknowledge their shortcomings and be lifelong learners so that they can best meet the needs of their students.

In addition to better understanding the content material, there are ways to teach probability while taking into account a group of diverse learners. As noted, activity based instruction is an exemplary way to teach probability. However, incorporating this into the classroom does not mean every student will succeed similarly. Although it has not been explored how varying students react to all areas of probability, student reactions when finding the probabilities for the sum of two dice have been examined (Nilsson, 2007). By using standard dice and also variations thereof, student problem solving when playing a game that was based on these sums was analyzed. There are a few ways the students could think about these sums. Specifically, they could think of possible or impossible events by finding the extremes. Moreover, they could think of possible or impossible events by finding the extremes and gaps created between those extremes by nonstandard dice. Finally, they could think of all of the combinations of the possible outcomes to determine what is more or less likely amongst them. Throughout a series of rounds, all students progressed through these thoughts linearly, although some groups reached conclusions faster than others. (Nilsson, 2007)

Nilsson (2007), thus concluded, that although the hands on and competitive aspect of this instruction was thought to be enticing for all students, the teacher still must be active in addressing learner variance. Supportive questioning was necessary to bring all students to critically assess the game according to all of the aspects described above. Moreover, it was noted that when students won the game, they did little self-reflecting because they did not think they needed to improve. Thus, guidance was needed to get these students to more carefully consider the probabilities at hand. (Nilsson, 2007) This adequately describes learner differences. Remedial students would require more intentional guidance in reaching the conclusions, while the advanced learners would require more encouragement to grow their minds and not just "get by" in their education, which is what they can become accustomed to doing (Tomlinson, The mobius effect, 2004). Teachers need to be deliberate in interacting with the students to ensure that they are all meeting the curricular goals and meeting additional expectations that are appropriate for individual students.

In addition to activities with intentional teacher support, there are other ways instruction can differentiate probability instruction. To do this, learning styles must be understood (Strong, et al., 2004). Those mentioned by Strong, et al. (2004) can all be addressed when teaching probability, which include mastery style, understanding style, interpersonal style, and self-expressive style. For the mastery style, students work by following steps, so for them, the rules of probability would be helpful. The understanding style is exhibited by students who find patterns and seek personal understanding of the material, ideal for the activity based instruction which has its foundation in the constructivist theory. Next, the interpersonal style is where students flourish in conversation with peers, once again, promoted by the group work required for activity based instruction. Finally, the self-expressive style is for those visual learners who prefer multiple representations of the material. The manipulatives such as dice, cards, and marbles used during probability are extremely advantageous for these learners (Dunn, 2005). (Strong, et al., 2004) Identifying different learning styles, abilities, and knowing how probability can be taught with respect to each of them, should encourage educators to instruct probability differentially.

Assessment

A major concern while differentiating is fair assessment. Novice teachers struggle to assess fairly when taking into account learner variance and are often hindered in differentiating because of that (Tomlinson, et al., 1997). Instead of being intimidated by what assessment should look like, teachers should be encouraged by what it can look like. Particularly, informal assessments hold tremendous potential (Allsopp, Kyger, Lovin, Gerretson, Carson, & Ray, 2008). By looking at the specific assessment, Mathematics Dynamic Assessment (MDA), Allsopp, et al. (2008) learned what a specific informal assessment can do to benefit the learning of mathematics and offered perspective for other similar assessments. Particularly, assessments should be relevant to student interests and experiences. For probability, that could mean connecting the homework and tests to the real world which simulate problems that deal with likelihood and chance. Moreover, questions should range from being concrete to abstract, both to assess what students are capable of in their thinking and to offer questions in which every student can answer at least one format correctly. It is also important to perform error pattern analysis to better understand and correct student misconceptions. The MDA guides teachers in this process but it is something that teachers should, and can, accomplish on their own. Finally, performing flexible interviews allow teachers to truly get to the heart of their students' understanding. This can be done by talking out problems with the students or by having the students observe the solving of a problem to find intentional mistakes. These nontraditional approaches to assessment reach those students who are interpersonal and allot teachers the opportunity to more deeply understand how their students are solving problems (Strong, et al., 2004). (Allsopp, et al., 2008) Therefore, teachers should be open to assessing students informally and recognize the benefits that it holds for a group of diverse learners.

An additional way to assess a diverse group of students is to use what has been called the interspersal technique (Skinner, Hall-Johnson, Skinner, Cates, Weber, & Johns, 1999). With this, student assignments include numerous target problems, interspersed with some easier problems. Research on this was done specifically with homework on multiplication, and it was shown that students would choose to do an assignment with more problems if it was interspersed as opposed to an assignment with fewer, but only the target problems. This was in spite of the fact that both assignments contained the same number of target problems. Furthermore, the students had better attitudes towards the homework with regards to the difficultly, their effort, and their time spent working on it. It is likely that confidence was built in doing the interspersed assignments which led to their completion and better attitudes. It is unknown whether the results would hold true if, after numerous assignment choices, the students recognized their differences. (Skinner, et al., 1999) Even though a causal relationship cannot be assumed, an optimism can be exhibited while offering choice on homework and interspersing problems.

Discussion and Conclusion

The literature offers the researcher a firm understanding of probability and differentiation. It is already known what hinders students in learning probability and what teaching methods can be used to overcome that. Specifically, activity based instruction, manipulatives, and peer interactions can be used to prevail over the misconceptions that result from the complexity of probability. Moreover, research has shown differentiation to be advantageous but also recognizes what stands in the way of its implementation. Novice teachers face many obstacles at the start of their career that cause them to resort to teacher centered instruction that does not take into account student variance. When teachers acknowledge, recognize, and act on the fact that the classroom is composed of diverse learners, learning is undeniably increased. However, in the research there is a glaring gap. No research was found for differentiating the proven teaching and assessment techniques for probability. Therefore, the researcher feels justified in combining these two aspects for a novel research study.

Moreover, all of the conclusions reached in the studies examined in the literature review cannot be automatically assumed for the classroom in which the researcher teaches. Numerous variations in environment, content, and student characteristics limit the transferability of the research. Therefore, in addition to combining what research has shown into a novel study, what has been shown elsewhere was also put to the test. Many of the research studies took place in other countries, causing one to question whether the same results would be true here in the United States. Even more specifically, varying settings such as rural, urban, or suburban, private or public schools, and regions of the United States, could cause varying findings. Thus, the researcher sought to understand the previous findings in the context of a rural, private school in the northeastern United States.

Chapter Three: Applications and Evaluations

Introduction

The objective of this research was to examine some implications of differentiation during a unit on probability. It was focused on seventh grade math students, while the sample of this population was students from a rural New York private school. Previous research on differentiation in the math classroom, particularly for probability, showed that it is beneficial for students. Moreover, research has shown that teaching probability with activities and experiments is advantageous. However, research has not been completed to show if differentiating activities along with instruction is similarly beneficial. Therefore, varied activity based instruction was utilized to instruct the students who were split into two groups based on their prior knowledge. By restricting the research to two differentiation groups, a small research sample, and one teacher, it was aspired that conclusive results could be reached for this small arena. Because of this, future probability instruction could be improved at this particular school.

Participants

All students taking the seventh grade math class at the rural New York private school were asked in class to participate in the research study. No incentive was offered to the students to encourage them to participate. Out of the eighteen seventh grade students, all students returned their consent forms, obtained parental consent, and consequently participated in the study.

Demographics for this class consisted of eighteen students whose average age was twelve years old. Because the school was a private school, the students came from a range of communities including, but not limited to, the suburbs of Henrietta and Victor and the rural towns of Lima, Honeoye Falls, and Avon. All of the students were American citizens whose first language was English. This is important to note because there were students who were adopted from the nations of Columbia and the Ukraine. Other than those two adoptees, all of the students were Caucasian.

Regarding the intellectual makeup of the students, they were a standard class. Throughout the school year, there were about five students who performed consistently at an above average level, applying advanced cognitive skills that allowed them to think about problems in a variety of ways. Next, there were about ten students who performed at a typical seventh grade level, making predictable mistakes but eventually mastering the required skills. Finally, there were about three students for whom math was not a subject that came easily to them but during the year they worked very hard to succeed. There were no students with a 504 plan but one with an IEP. This mandated a one-on-one aide to support him in his learning disability. Regardless, math was a strong subject for him and he was an average student.

Procedures of Study

After obtaining consent from the students and their parents, the students were allowed to choose a codename to put on their work for the remainder of the unit to maintain anonymity. Then they were given a survey to determine their perceptions of mathematics. This took less than a class period to complete and the remainder of the class period was used to further explain how the research period would proceed. The following day the students took a pretest from which the results were used to split the class into two groups, a lower differentiation (LD) group and an upper differentiation (UD) group. The test took the students one class period to complete and was used to determine their prior knowledge in probability. Once the test was scored, the students with the top half of the grades were placed in the UD group, with the other half composing the LD group. Students were unaware of how the groups were formed to maintain the integrity of the study. These two groups were further subdivided into groups of three students based on the researcher's knowledge of student dynamics. Halfway through the research period the groups of three students were shuffled, maintaining the distinctions of LD or UD, because some of the groups were not working well together. This switch also ensured that the students would not get suspicious of why they were in their particular groups. The researcher was initially surprised at what groups the students were placed into based on their prior knowledge. It was thought that this would be a mute point since the researcher thought that the class was obviously split into high and low halves. However, there were some students who unexpectedly tested into a group other than what the researcher tagged them to be in. This was an important reminder to keep an open mind with the students and to thoroughly assess their prior knowledge for each unit because teacher stereotypes on their intellect may not hold true when the content areas are different. Specifically, how the students performed in algebra may be different than their performance in geometry, and similarly different in probability.

Classroom instruction while the students were in these groups consisted of the students performing probability activities with manipulatives and experiments, and answering questions on a guided worksheet about the mathematics behind probability calculations. For the UD group, students were given limited guidance on classroom worksheets to encourage them to learn independently by making connections to what they already knew. For the LD group, students were offered more support on the classroom worksheets but they were still encouraged to build on their prior knowledge.

Every day, students were assigned practice homework that offered choices on which problems to complete and how many. Particularly, the day's topic was broken down into subsections, for which three problems of varying difficulty were offered. Students had to answer at least one question from each section. Homework was not graded but checked for completion of the requirement the day after it was assigned. The following day, the homework was gone over in its entirety with the entire class to ensure that no misconceptions ensued.

Once the unit was completed, the students retook the test from the beginning of the unit to determine how much knowledge was gained over the course of the unit and to compare them to last year's students who took the same test when their probability unit was not intentionally differentiated. Moreover, they retook the survey to assess if their perceptions towards mathematics were altered at all because of the differentiated, activity based instruction through which the probability unit was taught. Additionally, two focus groups of students from the LD and the UD were interviewed after school for more in-depth explanations.

Instruments for Study

This research utilized varying instruments to gather data results for quantitative analysis (Appendix A). One of these was the probability unit test administered as a pretest and a posttest. It was the same test that was administered to last year's seventh grade students so that when the study concluded, their scores could be compared. To determine if there was a significant difference between students who were taught probability through differentiated instruction and students who were taught without intentional differentiation, grades from the research group's unit posttest were collected and analyzed anonymously against last year's students who were not taught with intentional differentiation. Another unit was also statistically compared to ensure that there was not a significant difference between last year's students and this year's research group when they were taught in a similar manner. In addition, the pretest to posttest results for the class as a whole and between the two groups were compared and contrasted to determine the effectiveness of differentiation for increasing student cognition of probability concepts by targeting prior knowledge.

A survey was also administered before and after the probability unit by the researcher. The survey was modified from the Mathematics Attitude Inventory. It was altered to more pointedly reveal student perceptions of mathematics to determine if the instruction the students received altered those perceptions. Specific areas of investigation included self-concept in mathematics, enjoyment of mathematics, and motivation in mathematics. Self-concept is how competent students view themselves in specific areas (Trautwhen, et al., 2006), while enjoyment and motivation are self-explanatory. Furthermore, other extraneous information was gathered by the survey, such as gender and general perceived academic ability. Lastly, there was a section for comments where students candidly remarked on their attitudes towards the study and on mathematics class. Pre-survey and post-survey results were analyzed in each of the three categories for the entire class, and more specifically for the two subgroups of LD and UD.

In addition, two focus groups of students were randomly selected to be interviewed, one from the LD group and the other from the UD group. Interview questions were posed to gather more specific information about their attitudes towards math, the instruction they received, and their learning styles. The researcher met with these students after school and took typed notes while interviewing the students. These were later edited to create a more cohesive transcript.

Furthermore, the homework which was based on the foundation of choice was carefully tracked. Specifically, the quantity and quality of the problems selected for homework was taken into account to see if that information had a relationship with the type of instructional group the students were in or their achievement on the unit test. It was recorded how many problems the students attempted, how many of these were completed accurately, and which problem numbers constituted these two categories of information.

Chapter Four: Results

Control Test

Before analysis could be done to compare last year's seventh grade class' learning of probability with this years', a statistical analysis needed to be completed to ensure that there was no significant difference between them. The researcher proposed a null hypothesis that there would be no significant difference between unit test grades received by students from the two different school years when they were taught in a similar manner. The t-value, which represents the probability that there is no difference between the independent samples, was thus analyzed. Comparing it to a predefined t-value from the table at a 0.05 level allowed the researcher to reject or fail to reject the null hypothesis. **Table #1** shows the analysis of the control test along with the t-value.

Table #1

	2010 Control Test	2011 Control Test
Mean	89.62068966	83.72222222
Standard Deviation	9.99433337	13.37603456
Number	29	18
t = 1.612285		

The degrees of freedom was calculated to be 45, so from the table of t-values, the critical value at the 0.05 level was 1.960. Since the calculated t-value was less than the critical t-value, the researcher failed to reject the null hypothesis and concluded that there was no significant difference in the mean score of the two classes. This meant that the remainder of the data between the two years' classes could be statistically analyzed without adjusting the data to account for preexisting significant differences.

Prior Year to Posttest

Because there was no preexisting distinction between the two years' seventh grade classes, a statistical comparison of independent samples could be conducted on the probability test scores to conclude if there was a significant difference between them. This would lend information as to whether or not intentionally differentiating instruction was advantageous to student learning or not. The null hypothesis of this study was that there would be no significant difference between the probability unit test grades received by students who had their classroom instruction and assessment intentionally differentiated and those students who did not. **Table #2** reveals the result of the t-test so that the t-value could be compared to the t-value from the table at the 0.05 level.

Table #2

	2010 Probability Test	2011 Probability Test
Mean	88.86206897	64.61111111
Standard Deviation	12.87501495	12.96664902
Number	29	18
t = 6.249739		

Using the degrees of freedom of 45, the table of t-values revealed the critical value at the 0.05 level to be 1.960. Since the calculated t-value was more than the critical t-value, the researcher rejected the null hypothesis and concluded that there was a significant difference in the mean score of the two classes. It was evident that the means of the two years' probability tests were drastically different and not in favor of those students who had their instruction intentionally differentiated. There were many factors that played into these results, as will be expounded on in the Conclusions and Recommendations. Moreover, the test results were not the only factor in determining if differentiating the instruction and activities while teaching probability was advantageous. Therefore, this form of instruction should not be dismissed readily until the class' academic growth, student attitudes, and perceptions of mathematics are also taken into account.

Pretest to Posttest

Although compared to last year's class the probability test scores were significantly inferior, it was important to recognize this years' student growth in their cognition of probability. Therefore, a statistical analysis was done from the class' pretest scores to their posttest scores to determine if their development was significant. The null hypothesis was that there would be no significant difference between the pretest scores and the posttest scores. To determine this, a t-test for related samples was performed with the t-value displayed in **Table #3.**

Table #3

Pretest	Posttest	Diff	D^2
53	66	13	169
41	82	41	1681
59	71	12	144
12	62	50	2500
53	71	18	324
38	68	30	900
0	41	41	1681

	21	38	17	289
	65	74	9	81
	6	50	44	1936
	12	60	48	2304
	35	59	24	576
	12	62	50	2500
	38	56	18	324
	35	62	27	729
	41	80	39	1521
	41	79	38	1444
	41	82	41	1681
Sum:	603	1163	560	20784
Mean:	18	64.6111		
t = 14.0626				

From the table of t-values, the critical value at the 0.05 level was 2.110 when the degrees of freedom was 17. Since the calculated t-value was more than the critical t-value the researcher rejected the null hypothesis and concluded that there was a significant difference in the mean score of the class' pretest and posttest. Therefore, the change that occurred in the students' understanding of probability from the pretest to the posttest was significant. In fact, the t-value from pretest to posttest was greater than the t-value of the prior year's to this year's posttest. Since at this point it is impossible to determine what prior knowledge last year's class had in probability and how much knowledge they gained during instruction, it is plausible that this year had greater gains in their learning of probability than last year if last year's class had a lead on them in terms of prior knowledge. It is also plausible that last year's class started with comparable prior knowledge to this year's class and simply learned significantly more when taught without intentional differentiation. This quandary is impossible to settle without data on last year's class' prior knowledge, which the researcher does not have. Therefore, the researcher must be content to know that the differentiated approach to instruction did reveal significant gains in terms of student cognition which may or may not be the maximal gains that could be achieved depending on how the material was taught.

Test for LD to UD

It was essential to analyze whether those students with more or less prior knowledge of probability performed significantly different on the posttest as opposed to the pretest, as well. Statistical analyses were performed for both the students in the LD and the UD groups. The t-values from these groups and the calculations are in **Tables #4** and **Table #5** respectively. It was hypothesized that there would be no significant difference between the pre and posttest scores for either group.

Table #4

	Pre	Post	Diff	D^2
	12	62	50	2500
	0	41	41	1681
	21	38	17	289
	6	50	44	1936
	12	60	48	2304
	35	59	24	576
	12	62	50	2500
	38	56	18	324
	35	62	27	729
Sum	171	490	319	12839
Mean	19	54.44444		
t = 7.683402				

Table #5

	Pre	Post	Diff	D^2
	53	66	13	169
	41	82	41	1681
	59	71	12	144
	53	71	18	324
	38	68	30	900
	65	74	9	81
	41	80	39	1521
	41	79	38	1444
	41	82	41	1681
Sum	432	673	241	7945
Mean	48	74.77778		
t = 5.883301				

With the degrees of freedom now 8, the critical value to which the t-values were being compared was 2.306. For both the LD and UD groups, the t-value was more than the critical value so the researcher rejected the null hypothesis and concluded that there was a significant difference between the pretest and posttest scores for the LD and UD groups. As seen, the t-value for the LD group is greater than the t-value for the UD group. This implied that there was a greater significant difference in the scores for students in the LD group in the positive direction. Therefore, in terms of content knowledge growth, differentiation proved more advantageous for those students with less prior knowledge than for those with more.

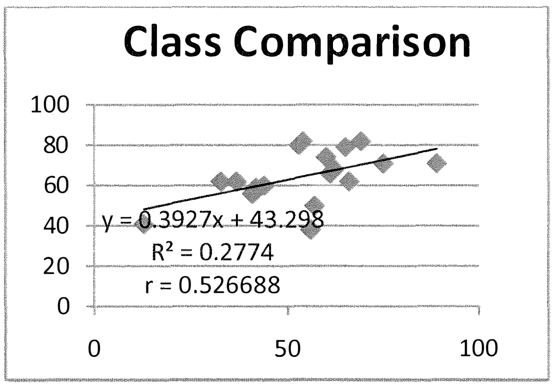
Homework Choice

Because the homework offered choices, it was interesting to see which students went above and beyond the requirement of answering only one problem per section. Therefore, for the LD and the UD groups it was calculated what percentage of students solved more problems than they had to. For both groups, there were nine students and six homework assignments, allocating fifty-four opportunities to surpass the requirement. Twenty out of these fifty-four opportunities were taken advantage of by the LD group; meaning 37% of the homework assignments completed surpassed the requirement. For the UD group, 57%, or thirty-one out of the fifty-four homework assignments, were completed beyond expectations. This had numerous implications, including the fact that the UD group required advanced differentiation because they were the students who excelled in math, so it was not surprising that they would go above and beyond requirements more so than the LD group. However, this was not without outliers. More than a third of the LD's homework was completed beyond the requirement, which showed that there were some students who wanted to practice the math concepts voluntarily. Moreover, there was one UD student, Zamboni37, who on two occasions did not even fulfill the requirement. Therefore, although there was a trend in homework completion when choice was offered, generalizations should not be overly relied upon.

Homework to Posttest

To efficiently examine homework's relationship to student learning as reflected by the posttest, an overall percentage was given for the students' homework completion. This was found by comparing the number of problems answered correctly by each individual student to the total number of problems attempted. More consideration will be given as to the content of the problems answered in the Conclusions and Recommendations section. These scores were then compared to the posttest scores to determine if there was a correlation between them. The r-value calculations are shown below for the entire class and the subsections of the LD and UD groups. These are then compared to the limits of an r-value, -1 to 1. It was hypothesized that there would be a weak correlation for all comparisons with r-values between -0.05 and 0.05. **Chart #1** shows the calculations for the r-value for the whole class' homework and posttest.

Chart #1



Because the r-value was greater than 0.05, the researcher rejected the null hypothesis and concluded that there was a significant positive correlation between the accurate homework completion and results on the unit posttest. That was not to say that homework completion caused corresponding test scores but rather there was a relationship between these two sets of information, to be more fully discussed in the Conclusions and Recommendations. Moreover, it was important to note that the correlation, although in the acceptable range, was still relatively weak compared to a perfect correlation of 1.

Charts #2 and **#3** display similar calculations for the LD and the UD groups, respectively.

Chart #2

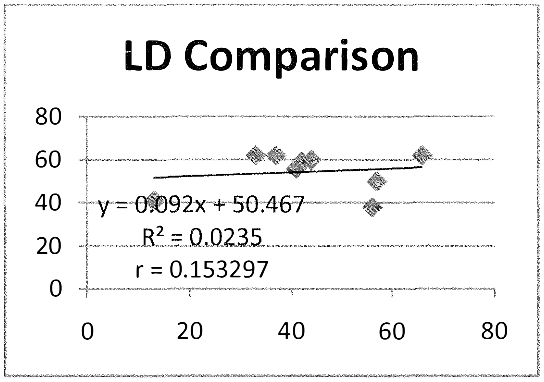
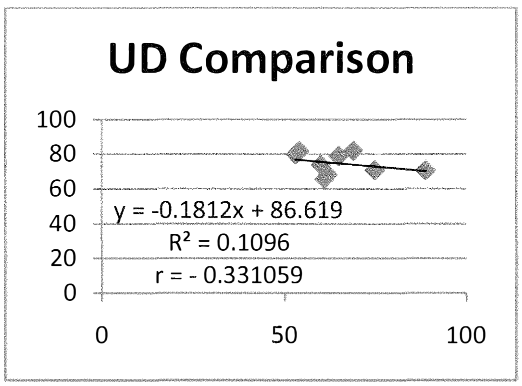


Chart #3



With both r-values being between -0.05 and 0.05, the researcher failed to reject the null hypothesis and concluded that homework averages and posttest scores were not significantly correlated for the individual differentiation groups. This implied that how accurately the students in each group completed their homework did not correspond with how well they did on the test. There were numerous implications that this held, to be more elaborated on in the Conclusions and Recommendations.

Survey to Survey

The survey assessed three particular domains when it came to student perceptions in mathematics; self-concept, enjoyment, and motivation. The survey was administered before the research period and after to determine if differentiating instruction significantly influenced student beliefs in any of these three areas. The null hypothesis was that there would be no significant difference between the survey results in any of these three areas at a 0.05 level. Three t-tests were performed to determine the validity of that hypothesis.

With regards to self-concept in mathematics, student scores pre and post-research were statistically analyzed. These scores were found by using a defined formula with the ratings assigned to each related question. Note that the scores in this category could range from 6 to 28, with the lower scores revealing greater self-concept. The t-value for related samples was calculated as shown in **Table #6.**

Table #6

	Pre	Post	Diff	D
	12	14	2	4
	7	7	0	0
	18	17	-1	1
	14	14	0	0
	15	13	-2	4
	10	9	-1	1
	22	22	0	0
	10	12	2	4
	7	7	0	0
	21	20	-1	1
	18	20	2	4
	16	19	3	9
	10	11	1	1
	13	14	1	1
	8	7	-1	1
	11	12	1	1
	10	8	-2	4
	6	10	4	16
Sum:	228	236	8	52

Mean:	12.66667	13.11111
t = 1.117008		

From the table of t-values, the critical value at the 0.05 level was 2.110. Since the calculated t-value was less than the critical t-value the researcher failed to reject the null hypothesis and concluded that there was no significant difference in the mean self-concept scores from the survey pre and post-research. Significant or not, the self-concept mean did go up for the post-research surveys revealing that the students had an insignificant decrease in their self-concept of mathematics, based on what the score signified as previously defined.

The range for the enjoyment of mathematics survey scores was 7 to 28, once again lower scores signifying greater enjoyment. **Table #7** shows the statistical calculations for the t-value for comparing pre and post-research enjoyment of mathematics.

Table #7

	Pre	Post	Diff	D^2
	11	12	1	1
	10	8	-2	4
	17	19	2	4
	16	18	2	4
	17	17	0	0
	19	17	-2	4
	23	25	2	4
	16	18	2	4
	7	8	1	1
	24	24	0	0
	23	23	0	0
	14	19	5	25
	15	16	1	1
	21	26	5	25
	13	17	4	16

	16	11	-5	25
	16	10	-6	36
	7	7	0	0
Sum:	285	295	10	154
Mean:	15.83333	16.38889		
t = 0.797639				

From the table of t-values, the critical value at the 0.05 level was 2.110. Since the calculated t-value was less than the critical t-value the researcher failed to reject the null hypothesis and concluded that there was no significant difference in the mean enjoyment scores from the survey pre and post-research. As with self-concept, the mean score went up which signifies an inconsequential decrease in enjoyment.

Finally, the third part of the survey to be analyzed was student motivation in mathematics. Scores on this section could range from 4 to 16, with the lower scores depicting the more self-proclaimed motivated students. Calculations for the t-value for this portion of related samples are in **Table #8**.

Table #8

	Pre	Post	Diff	D^2
	9	10	1	1
	9	10	1	1
	14	13	-1	1
	12	12	0	0
	16	12	-4	16
	10	10	0	0
	16	16	0	0
	13	11	-2	4
	7	9	2	4
	16	16	0	0
	15	15	0	0
	12	13	1	1
	12	11	-1	1
	16	16	0	0

	12	9	-3	9
	10	11	1	1
	11	9	-2	4
	9	9	0	0
Sum:	219	212	-7	43
Mean:	12.16667	11.77778		
t = 1.071898				

From the table of t-values, the critical value at the 0.05 level was 2.110. Since the calculated t-value was less than the critical t-value the researcher failed to reject the null hypothesis and concluded that there was no significant difference in the mean scores from the survey pre and post-research. Conversely to the other two categories, there was an insignificant improvement in motivation as the mean score decreased. This can be attributed to the nature of the research where the students were in self-directed groups doing exploratory learning with the guided worksheets and experiments. In order to learn the material, it was essential that they were engaged in the classroom activities and being self-advocates, asking for assistance when needed. Because of this, the students may have perceived more self-motivation with regards to their education, albeit insignificant.

All of the changes on the survey that occurred pre to post-research were shown to be insignificant. Therefore, the research neither benefited nor harmed the students' perceptions of mathematics in any way that is worth mentioning. However, this was with regards to the class as a whole. Therefore, to be more thorough, the LD and the UD groups' data should be independently compared.

Survey for LD and UD

It is important to investigate whether those students with more or less prior knowledge of probability had their perceptions of mathematics changed significantly because of the research performed. Statistical analyses were performed for all three categories of the survey for both the students in the LD and the UD groups. The t-values from these and the calculations are in **Table #9**. This time the critical t-value to which the data was being compared was 2.306 because the degrees of freedom was 8. Abbreviations in the table include, SC being self-concept, E being enjoyment, and M being motivation.

Table #9

LD SC	Pre	Post	Diff	D^2
	14	14	0	0
	22	22	0	0
	10	12	2	4
	21	20	-1	1
	18	20	2	4
	16	19	3	9
	10	11	1	1
	13	14	1	1
	8	7	-1	1
Sum:	132	139	7	21
Mean:	26.4	27.8		
t = 3.011976				

UD SC	Pre	Post	Diff	D^2
	12	14	2	4
	7	7	0	0
	18	17	-1	1
	15	13	-2	4
	10	9	-1	1
	7	7	0	0
	11	12	1	1
	10	8	-2	4
	6	10	4	16
Sum:	96	97	1	31
Mean:	19.2	19.4		
t = 0.305348				

LD E	Pre	Post	Diff	D^2
	16	18	2	4
	23	25	2	4
	16	18	2	4
	24	24	0	0
	23	23	0	0
	14	19	5	25
	15	16	1	1
	21	26	5	25
	13	17	4	16
Sum:	165	186	21	79
Mean:	33	37.2		
t = 6.506612				

UD E	Pre	Post	Diff	D^2
	11	12	1	1
	10	8	-2	4
	17	19	2	4
	17	17	0	0
	19	17	-2	4
	7	8	1	1
	16	11	-5	25
	16	10	-6	36
	7	7	0	0
Sum:	120	109	-11	75
Mean:	24	21.8		
t = 2.379333				

LD M	Pre	Post	Diff	D^2
	12	12	0	0
	16	16	0	0
	13	11	-2	4
	16	16	0	0
	15	15	0	0
	12	13	1	1
	12	11	-1	1
	16	16	0	0
	12	9	-3	9
Sum:	124	119	-5	15
Mean:	24.8	23.8		
t = 2.42712				

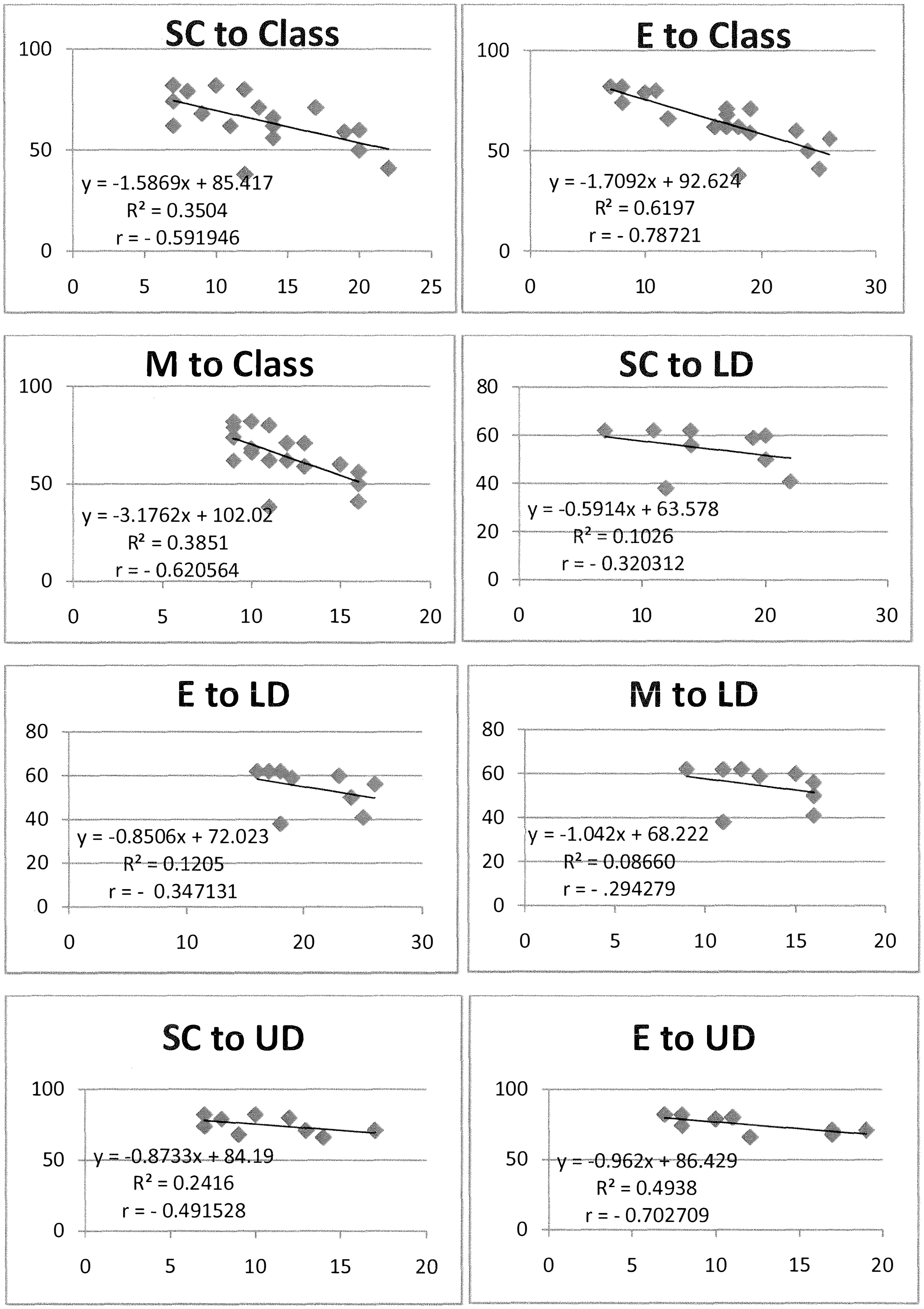
UD M	Pre	Post	Diff	D^2
	9	10	1	1
	9	10	1	1
	14	13	-1	1
	16	12	-4	16
	10	10	0	0
	7	9	2	4
	10	11	1	1
	11	9	-2	4
	9	9	0	0
Sum:	95	93	-2	28
Mean:	19	18.6		
t = 0.646579				

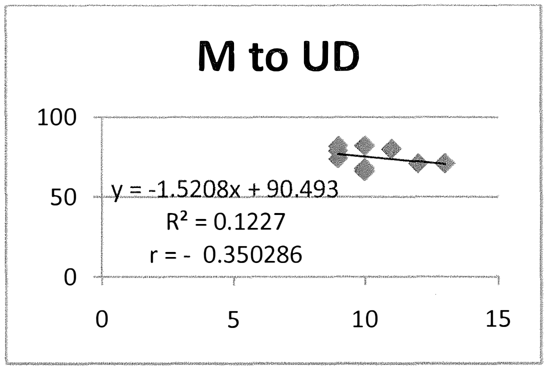
As seen by the t-values in the table, breaking down the data to individually look at the UD group and the LD group did reveal some significant differences. Therefore, the researcher rejected the null hypothesis for the UD group in terms of enjoyment and for the LD group in terms of self-concept, enjoyment, and motivation. For the UD group, their enjoyment significantly improved. This can be attributed to the fact that because they are good in math, being challenged by the instructional methods brought them even more satisfaction with the subject. The LD group saw significant differences in their opinions prior to and after the research period; however, in two out of the three categories it was for the worse. The LD group decreased in their self-concept of mathematics and in their enjoyment of it. While it was thought that the engaging activities would encourage the lower achieving students to discover more math concepts on their own and boost their confidence, it was actually more discouraging to their perceived abilities than when they were taught with primarily traditional lecture methods. Moreover, the group work was proposed to make the learning experience more exciting for the students but the LD group apparently found it less enjoyable than the instruction they were receiving previously. Fortunately, the LD group was significantly more motivated in math class presumably for the same reasons noted for the class as a whole.

Survey to Posttest

Another area of correlation to be examined was whether student attitudes as reflected on the survey were related to the students' scores on their posttest. Because there were three specific portions of the survey, nine analyses were performed. Particularly, self-concept, enjoyment, and motivation survey scores from the concluding survey were compared to the posttest scores for the whole class, the LD group, and the UD group. The null hypothesis was that there would be a weak correlation for all comparisons defined by r-values being between -0.05 and 0.05. **Chart #4** shows the calculations for the r-value for all of the above described comparisons.

Chart #4


SC to Class
y = -1.5869x + 85.417
R² = 0.3504
r = - 0.591946
E to Class
y = -1.7092x + 92.624
R² = 0.6197
r = - 0.78721
M to Class
y = -3.1762x + 102.02
R² = 0.3851
r = - 0.620564
SC to LD
y = -0.5914x + 63.578
R² = 0.1026
r = - 0.320312
E to LD
y = -0.8506x + 72.023
R² = 0.1205
r = - 0.347131
M to LD
y = -1.042x + 68.222
R² = 0.08660
r = - .294279
SC to UD
y = -0.8733x + 84.19
R² = 0.2416
r = - 0.491528
E to UD
y = -0.962x + 86.429
R² = 0.4938
r = - 0.702709



There were some comparisons with an r-value that caused the researcher to reject the null hypothesis and assume a significant correlation. These were all of the survey categories when compared to the entire class' posttest grades and the comparison of the enjoyment survey scores to the UD group. The remainder of the comparisons for correlation had an r-value which the researcher used to fail to reject the null hypothesis and conclude that the correlation was insignificant. For significant correlations, it is important to note that the r-values were all negative. This implies that as one set of data went up, the posttest scores, the other set of numbers was going down, the survey scores. However, recall that for the survey scores, a lower score held better implications for the three areas. Therefore, the correlation is genuinely implying that a better student perception in each of the three areas was related to a better posttest score. For the entire class, this correlation was significant for all three categories. Therefore, for the class as a whole, it can be affirmed that the post-survey attitudes of the students had a relationship to their posttest score. Reasons for this are elaborated on in the Conclusions and

Recommendations. Similar discussion will result for the relationship between the enjoyment survey results and the posttest scores for the UD group.

Chapter Five: Conclusions and Recommendations

The results of the research have brought about numerous points of discussion. To begin, a major significant difference was found between last year's and the research year's probability posttest results. The difference in the mean scores of these exams was significantly different, and not in favor of the research students. Specifically, the mean score for last year's class was an 89, while the research group had a mean score of 65. This does not bode well for the form of instruction this research was analyzing. It was already noted that this cannot be taken at face value in the Results section, particularly because the previous year's prior knowledge was not taken into consideration. Therefore, a recommendation for future research would be to take into account both classes' prior knowledge via a pretest for a more accurate comparison of their learning through a posttest.

However, there are numerous other factors that the researcher would like to take into consideration before immediately dismissing the differentiated instruction used during the research period. To begin, this year's class had a couple of factors that quite likely influenced their performance on the posttest. First of all, the probability unit was taught extremely close to the end of the year. Although probability is traditionally taught as the last unit of the year, this year it was taught within the last couple of weeks of school. Reasons for this were unavoidable and unforeseen as the students had field trips and school assemblies. Last year's class used those final weeks to review for their final exam, with the probability unit being taught prior to that. This closeness to the end of the school year, and consequently summer vacation, resulted in decrease focus and drive on the part of the students, as observed by the researcher. Consequently, their learning of probability was diminished, although, it should not have been to the degree revealed by the statistical analysis. Another factor that may have contributed to this significant difference in test scores was the fact that the students did not review for the test. Throughout the entire year, the students always had a review day before a test. However, because of the time constraints described above, the research students were not allocated this review. This change in routine could have accounted for some of the poor test grades. Thus, for future research it should be ensured that no other educational factors are altered besides those intended for the study.

The aforementioned reasons for the negative significant difference tend to defend the research. However, the results were so negative that there needs to be some accountability for the construct of the study. A point of issue for the differentiated instruction is that the students were not prepared for the intense problem solving required by the unit. As mentioned in the methodology, the students were to perform probability activities to complete guided worksheets which were designed to require more or less prior knowledge depending on the group. However, throughout the year and presumably most of their educational careers, mathematics instruction for the research students had been traditional and teacher based with the teacher explaining a concept, performing examples, and allowing the students practice time. At this point in their education, changing instructional methods was not something that could be expected to go over flawlessly. It was evident that the students struggled with this transition and thought at times that they were not being instructed efficiently or that they were incompetent because they did not fully understand the probability topics. The two interviews, LD and UD, in Appendix B, offered student responses that affirmed this struggle. Students in both groups affirmed added confusion because of the research construct. In the LD group, Wander was the most negative of the study stating, "The probability thing for school [graduate school] has made it harder. I feel like I haven't learned anything in this unit because we had to teach ourselves how to do it and I didn't have any idea how to do it." It was unfortunate that she felt this way towards the exploratory learning but it is also understandable based on the class' educational history. In the UD group, a student offered constructive criticism of the research by saying, "I think a little more explanation from you would have been helpful to end class." Although the UD group was not as negative in the interviews of the differentiated instruction methods, at least one student still felt as though concepts could have been made clearer at the end of class. Once again, this was probably largely in part because the students were insecure in this novel form of learning. A recommendation for future research is that the differentiated instructional methods be used for a few subsequent units so that the students could adapt to activity based exploratory instruction and the results of it could be more accurately assessed.

Even though the posttest results compared to the previous year were significantly negative, the statistical analysis did show notable gains for the research group from the pretest to the posttest. In the Results sections, the ramifications of this finding were discussed. More specifically, the LD may have shown improvement because, for some students, the break from the norm worked for them. Even though some of the students were verbal in commenting on their disapproval of the study, unbeknownst to them this change from traditional instruction potentially jolted their system for them to make great strides in their learning without them even being aware of it. For the UD group, the students could have had growth in their knowledge because these were the students for whom school is designed for; particularly, they knew how to function in the system. For them, the change in instructional methods was just another framework for them to learn to function in and when they promptly did, they were able to thrive in it.

Conversely, it was not elaborated on as to why there was a greater significant difference for the LD group as opposed to the UD group. It was surprising to have this result based on the interview and survey results which revealed that the students in this group felt like they had learned very little. In the LD interview (Appendix B), Rori remarked that, “I never understood what we were doing until after the fact and even then I’m not sure if I did.” This can be contrasted with some of the comments on the survey from the UD group and how, even though the research was far from perfect, the students were not as intimidated by the instruction methods. Appendix C shows these positive comments. Moreover, the survey comments affirmed the UD groups’ view of the importance of mathematics. Therefore, however unexpected the LD students would affirm their learning was, it can be rationalized why they had a greater significant difference than that UD group. It can be related to the fact that the students were not used to this instructional style and were insecure because of it. Since they were unconfident in the instructional methods, their perceived learning did not match what was demonstrated on the test. Regardless, based on its nature and what previous research has shown, it was not surprising that differentiated instruction significantly improved student learning from the pretest.

Moreover, the pretest may not have accurately revealed prior knowledge. Because the LD students were characteristically low achievers, they may have felt that the pretest was pointless; more specifically, they would not know anything if they had not yet learned it. This was evidenced by Criminal's test (Appendix D) when she did not even try. Therefore, when they took the posttest the results could have been significantly different because the effort put forth increased when the LD students felt like they could actually complete what was being asked of them. This presumably caused their scores to increase from scores that were already lower than necessary since a lack of confidence could have resulted in a lack of performance. Although the UD group also had their learning significantly increased, the t-value was potentially not as high because they had greater prior knowledge. Because the students in the UD group knew more about probability entering the unit, they may have not have shown as much improvement because they did not have as large a margin to improve.

In addition to the statistical analysis of homework described in the Results, specific homework questions should be discussed as they relate to the research.

Appendix E contains the table showing the quantity and quality of homework questions answered. As seen, there was a pattern to problems answered and omitted by the students. Particularly, students were more likely to answer the more straightforward questions than the more in depth questions which required greater problem solving. This was not surprising given that the population was seventh grade students nearing the end of their school year. Exemplification of this was on Homework #2, as every student answered problem one and on Homework #6, when every student, save one, answered problem one. Examples of student work on these two assignments are in Appendix F. For Homework #2, it was apparent that problem one was easier than the others in that section simply because the students did not have to toss a coin as many times. For Homework #6, problem one required using the least number of people to form combinations and was notably similar to a problem from the guided notes. It makes sense that the other problems in that section would not be attempted as readily, although some did if they chose to complete more than the minimum one problem per section. There are various other examples of homework problems that were answered more frequently than others. They need not be discussed at length since they align with the analogous reasons of them being easier to answer based on what was directly covered during instruction or on the amount of work required to answer them.

Additionally, it is important to note which problems were omitted entirely by the class as a whole. These include Homework #3: problem eight, Homework #4: problem eight, and Homework #6, problem fifteen. Examples of homework with their omissions are in Appendix G. They revealed apparent reasons as to why students would skip these questions. Homework #3 was a contextual problem different from what they were used to working with in class. The other two problems in that section dealt with manipulatives the students used to perform their differentiated experiments during class. The omitted problem was on selecting people and that difference from what the students were used to in examples explains why it would be skipped by everyone. Homework #4 had a problem omitted for less obvious reasons. The questions before and after it in the section were very similar and related to the same context. Therefore, the researcher believed that it was omitted because it was the hardest to discern as to what was being asked, the colors of the marbles were neither all different nor all the same. Finally for Homework #6, the skipped problem required the students to create their own problem both in terms of permutations and combinations and solve it both ways. Because this problem required so much work on the part of the students and a thorough understanding of the distinction between permutations and combinations, it was understandable why the students would not want to complete this problem.

When comparing the homework percentages to posttest results, there was a significant correlation. As already cautioned, this should not be confused with causation, or that higher homework percentages caused higher posttest scores. Regardless, there were numerous rationales for the relationship between these two sets of data. The most obvious of which was that the students who did better on the homework, grasped the concepts better initially, and were consequently better able to apply that understanding to perform on the posttest. Furthermore, the relationship may reveal the internal drive of the students. Those students who answered more questions accurately on the homework may have done so because they sought help either from parents or the researcher to complete it, and those same self-motivated students could have used study skills to prepare for and eventually perform comparably on the test. These same explanations could be applied to students on the lower performing end of the spectrum by applying reverse reasoning.

On the other hand, when the LD and UD group's posttest scores were compared to homework performance, a similar correspondence was not found. This could largely be attributed to the fact the correlation coefficient for the entire class' comparison was deemed significant but was still relatively weak. Therefore, when dividing the class into the two research groups for comparison, the connection between the two sets of data was lost. In fact, the same reasons that affirmed the relationship for the entire class most likely were lost as the high and low students were not balancing each other out to emphasize the relationship between homework performances and test scores.

The survey comparisons held numerous points of discussion, a few of which were proved significant by statistical analysis. Particularly, there was a significant difference for the UD group in terms of enjoyment and for the LD group in terms of self-concept, enjoyment, and motivation. It was apparent why the UD group would show improved enjoyment of mathematics because of the differentiated instructional methods. These students were the ones who generally performed well in mathematics and understood the rich complexities of the topic. This form of instruction allowed these students to be challenged and to work with similar peers to reach conclusions. It is likely that traditional classroom instruction would get boring for these students who usually understand concepts upon their first explanation but still have to listen to subsequent explanations directed to the rest of the class. Consequently, it was understandable as to why this form of instruction would be more enjoyable for them.

Alternatively, the LD group decreased in their self-concept of mathematics and their enjoyment of it. This was largely in part due to their perceived knowledge learned and the means of its instruction during the research period. As for the decrease in self-concept, this can be attributed to the construct of the differentiated instructional methods. As noted in the survey, the students seemed to feel incompetent in learning math via the methods prescribed for the research. This was because they were doing exploratory learning and when they did not fully understand what they were doing, consequently felt incompetent. Feelings of incompetency were evidenced in Appendix B. Although, the pretest to posttest statistical analysis did reveal significant gains for the students in their learning of probability, this was not what they felt with regards to their ability in mathematics. Regardless, their adamantly stated opinions of the research would obviously influence their self-concept in mathematics. Moreover, it was understandable why the students would have a decrease in their enjoyment of mathematics. If they were not feeling as if they were learning or that they possessed the innate ability to learn mathematics, they were not going to enjoy it. Nevertheless, this was somewhat surprising as it was anticipated by the researcher that the group work and activities would be stimulating and enjoyable for all students. Apparently the objectives of the class, learning probability concepts, trumped the actual mode of learning as portrayed by the survey results. In the future, it is recommended that the LD students be supported more by a teacher during differentiated instruction so that they do not feel so incapable and to maintain an enjoyment of learning. This could be done by altering the guided worksheets to provide more information to the students on the mathematics patterns they were seeking to uncover. Additionally, the teacher could moderate a summary discussion at the end of class for all of the groups to discuss what they did not understand that day to rectify those knowledge gaps or misunderstandings.

Fortunately, there was one arena in which the LD group increased over the course of the research and that was their motivation. This was most likely because the research required every student to participate in the probability activities and to draw on their prior knowledge to construct their novel understandings. Because the differentiated instruction required this intentionality, it is likely that they perceived increased motivation because of the amplified effort they put forth.

Finally, the survey was correlated to the posttest and there was a significant correlation coefficient for the comparisons of all of the survey categories compared to the entire class' posttest grades and the comparison of the enjoyment survey scores to the UD group. Once again, the survey scores cannot be assumed to have caused the test grades but there are some reasons for the significant relationship. For the entire class, it is likely that the high and low ratings for self-concept, enjoyment, and motivation were able to be correlated to the high and low test grades. It appears as though a balance was found which caused the correlation. It is probably true that there was some relationship between students with low self-concept and those students who actually did not perform very well on the posttest, and vice versa. Similar logic can be applied for those students who did not enjoy the instructional methods or who were less motivated and their test scores, and vice versa. When the statistical analysis was performed more specifically, there was only a significant correlation for the UD group and their enjoyment. When the balance between high and low students was stripped away, it was not as blatant a relationship between survey scores and posttest grades. However, the UD group did maintain this distinction between their enjoyment and test scores. This shows that those students who performed very well on the posttest also enjoyed the instructional methods very much, while those students who did not do well on the test enjoyed the mathematics less. Confirmation of this is straightforward since the UD group had the most significant improvement in their enjoyment of mathematics pre to post-research. It makes sense that this strong factor would correlate to the test grades of the higher achieving students.

Research on differentiated instruction revealed a great deal about its implications for a seventh grade probability class at a rural New York private school. Although it did not seem entirely beneficial when the posttest scores were compared to last year's class and for a couple of the survey categories for the LD group, future research could see different results if some of the aforementioned recommendations are taken into consideration. By doing this, the research would better align with the optimistic results revealed in the literature review.

However, in spite of the research being negative in some regards compared to prior research, there were still some benefits revealed by the research. Some student attitudes were changed for the better as revealed by the survey, and learning did occur for the students over the course of the unit. It would be an interesting study to see if the knowledge gained is actually retained and more meaningful to the students in the long run because of the way in which the students constructed it with their prior knowledge. Furthermore, the students benefited from being shown a novel form of instruction that exposed them to more problem solving and required all students to be engaged in the learning process at all times. Another study that would reveal more accurate implications of this form of instruction would be to utilize similar instruction over the course of a few units so that the students could adjust to the change in their routine and better function in their learning environment.

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Appendix A

Probability Test

Name: ____ Date:________________

Answer the following problems, showing all necessary calculations.
Unless specified, write probabilities as fractions.

1. Six balls numbered from 1 to 6 are placed in an urn. One ball is selected at random. Find the probability that it is NOT number 5.

2. A single fair number cube is tossed. Find the probability of obtaining a number other than 9.

3. There are 10 crayons in a box. Two are orange. A crayon is selected at random. Find the probability that it is orange as a fraction, decimal, and percent.

4. The table shows the drink preferences of 50 shoppers at the mall. What is the probability that one shopper, selected at random from the 50 surveyed, preferred either Drink A or Drink C?

Drink Survey

Drink	Number of Shoppers
A	12
B	7
C	14
D	6
E	11

5. From a barrel of colored marbles, you randomly select 7 blue, 5 yellow, 8 red, 4 green, and 6 purple marbles. Find the experimental probability of selecting a marble that is NOT purple.

6. A Lights-A-Lot quality inspector examines a sample of 25 strings of lights and finds that 3 are defective. What is the best prediction of the number of defective strings of lights in a delivery of 150 strings of lights?

7. A bag contains 2 purple marbles and 4 green marbles. One marble is drawn and not replaced. Then a second marble is drawn. What is the probability that the first marble is green and the second one is purple?

8. A bag contains five blue marbles and one white marble. The marbles are randomly selected one at a time. What are the odds of picking the white marble on the first selection?

9. A multiple-choice test has 5 questions, each with 4 possible answers. Find the probability of guessing the correct answers to all of the questions.

10. A car manufacturing makes three cars: a hatchback, a coupe, and a sedan. These cars can come in three different colors: silver, grey, and black. Make a tree diagram to show the sample space for the different selections. What is the probability of randomly selecting a yellow coupe?

11. Use the counting principle to determine how many different meals can be made from 3 beverages, 4 sandwiches, 3 sides, and 2 desserts.

12. How many different ways can you arrange five people shoulder-to-shoulder in a line?

13. There are 14 students participating in a spelling bee. How many ways can the students who go first, second, third, fourth and fifth be chosen?

14. Four cards are drawn in succession, without replacement, from a standard deck of 52 cards. How many sets of four cards are possible?

15. A committee is to have three members. There are seven men and four women available to serve on the committee. How many different committees can be formed?

Mathematics Attitude Survey

Please respond to the following questions before going on to the Mathematics Attitude Survey on the back. Check the appropriate space for each item.

a. Research Group: _____ A _____B

b. Gender: _____ Male _____ Female

c. What grades do you normally receive in school? *Check only one response*

____ Mostly A's ____ Mostly A's and B's
____ Mostly B's ____ Mostly B's and C's
____ Mostly C's ____ Mostly C's and D's
____ Mostly D's

Please mark the box that best matches your opinion.
(SD = Strongly Disagree, D = Disagree, A = Agree, SA = Strongly Agree)

Statement	SD	D	A	SA
1. Mathematics is something which I enjoy very much.				
2. I do not do very well in mathematics.				
3. Doing mathematics problems is fun.				
4. I would like to do some outside reading in mathematics.				
5. Mathematics is easy for me.				
6. When I hear the word mathematics, I have a feeling of dislike.				
7. I would like to spend less time in school doing mathematics.				
8. Sometimes I read ahead in our mathematics book.				
9. I usually understand what we are talking about in mathematics class.				
10. No matter how hard I try, I cannot understand mathematics.				
11. I would like a job which does not use any mathematics.				
12. I enjoy talking to other people about mathematics.				
13. I am good at doing mathematics problems.				
14. Sometimes I do more mathematics problems than are given in class.				
15. I remember most of the things I learn in mathematics.				
16. I have a good feeling towards mathematics.				
17. I have a real desire to learn mathematics.				

Other Comments:

Interview Questions

1. What was your opinion of class during the research period?

2. What was the opinion of your class group?

    a. Would you have chosen to be in the group you were assigned or the other group? Why?

3.
    a. What was your opinion of math before the research period?

    b. What is your opinion of math now?

4. What would you have changed about class while learning probability?

Day 1: Probability LD

1. Play the game two times!

Write down your personal strategies after playing the first game:

Write down your group's strategies after playing the second game:

What do you think probability is?

Why do we learn about it?

2. Words to know (pg. 629):

Outcome –

Event –

Theoretical Probability –

Complement –

Odds in Favor –

Odds Against –

3. Let's do the math.

What is the lowest number used to represent a probability (when something is impossible)?
Give an example of an event with this probability.

What is the highest number used to represent a probability (when something is certain)?
Give an example of an event with this probability.

How is probability calculated (it's one of the definitions above)?

Based on the definition, probability is a [whole number, fraction, irrational number].

Earlier this year we learned to rewrite these as decimals and percents. We can also represent probabilities in those forms.

4. Examples (Use your supply basket if necessary)

What is the probability of rolling an even number on a standard die? {Notation: P(even number)}
Write all three ways.

What is the probability of *not* drawing a heart from a standard deck? Write as a fraction.
What do we call this type of problem?

Odds: Write the number described and the number representing the rest of the group, separated by a colon. For example, the odds in favor of getting a head when flipping a coin is 1:1 (one head: one tail)

What are the odds in favor of rolling a 2 on a standard die?

What are the odds against drawing a 4 from a standard deck?

Day 1: Probability UD

1. Play the game two times!

Write down your personal thoughts after playing the first game:

Write down your group's thoughts after playing the second game:

What is probability?

Why do we learn about it?

2. Words to know (pg. 629):

Outcome –

Event –

Theoretical Probability –

Complement –

Odds in Favor –

Odds Against –

3. Let's do the math.

What is the lowest number used to represent a probability?
Give an example of an event with this probability.

What is the highest number used to represent a probability?
Give an example of an event with this probability.

How is probability calculated?

Based on this, probability is a ___________________.

Earlier this year we learned to rewrite these as decimals and percents. We can also represent probabilities in those forms.

4. Examples (Use your supply basket if necessary)

What is the probability of rolling an even number on a standard die? {Notation: P(even number)}
Write all three ways.

What is the probability of drawing a heart from a standard deck? Write as a fraction.

What is the probability of *not* drawing a heart from a standard deck? Write as a fraction.

What do we call this type of problem? Do you see the relationship? Why?

Odds: Write the number described and the number representing the rest of the group, separated by a colon.

What are the odds in favor of rolling a 2 on a standard die?

What are the odds against drawing a 4 from a standard deck?

Homework #1

Name: ______________________________________Date: ___________________

Please complete at least one problem from each section. Unless specified, write probabilities as fractions.

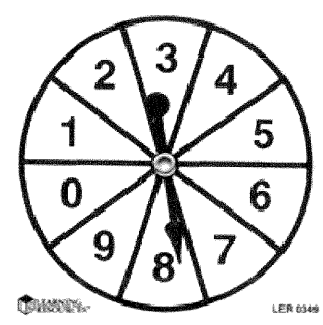
Section 1:

1. You mix the letters [A, Q, U, A, I, N, T, A, N, C, E] thoroughly. Without looking you draw one letter. Find the P(vowel). Write the probability as a fraction, decimal, *and* percent.

2. You mix the letters [M, I, S, S, I, S, S, I, P, P, I] thoroughly. Without looking you draw one letter. Find the P(S). Write the probability as a fraction, decimal, *or* percent.

3. Write down a word of your choice. Find the P(consonant). Write the probability as a fraction, decimal, and percent.

Section 2:



Suppose you spin the spinner once.

4. P(12) =

5. P(2 or 4) =

6. P(number less than 5) =

Section 3:

You roll a number cube with the numbers 1-6. Find the odds in favor of the outcome.

7. Rolling a 5

8. Rolling a 3

9. Rolling an odd number

Section 4:

The U.S. House of Representatives has 435 members. Suppose each members name is put into a hat and one name is chosen at random to have dinner with the president. Find the probability a person from the given state is chosen as a decimal rounded to the nearest hundredth.

State	Number	State	Number
Florida	27	New York	27
Pennsylvania	18	California	53
Colorado	7	Texas	36

10. P(New York)

11. P(Colorado)

12. P(California)

Section 5:

13. There are six chemical elements called "noble gases." Suppose you write the names of all 112 elements on cards and select a card without looking. What is the probability of *not* picking a noble gas? Write the probability as a fraction, decimal, *or* percent.

14. The odds in favor of an event are 5 to a. What is the probability of the event in terms of a?

15. Suppose you have 3 nickels, 3 dimes, and 3 quarters in your pocket. Does the probability of drawing a dime from your pocket equal the probability of drawing a quarter?

Day 2: Experimental Probability LD

1. Understanding meaning

What do you think experimental probability is?

Look it up! (pg 636)

2. Explore

What is the theoretical probability of drawing a club from a standard deck of cards?

Use the deck of cards from your supply basket and keep track of your information in the following table.

Club	
Spade	
Heart	
Diamond	

Draw a card and replace it ten times. Using that information, what was the probability you drew a club?

Draw a card and replace it fifteen more times. Using the compiled information, what was the probability you drew a club?

Draw a card and replace it twenty more times. Using the compiled information, what was the probability you drew a club?

What happened to your experimental probability, when comparing it to the theoretical probability, as you increased the number of times you drew a card?

Using your last experimental probability, if you were to draw and replace a card 100 times, how many cards would you expect to be clubs?

Day 2: Experimental Probability UD

1. Understanding meaning

What is experimental probability?

Look it up! (pg 636)

2. Explore

What is the theoretical probability of drawing a club from a standard deck of cards?

Use the deck of cards from your supply basket and keep track of your information in the following table.

Club	
Spade	
Heart	
Diamond	

Draw a card and replace it ten times. Using that information, what was the probability you drew a club?

Draw a card and replace it fifteen more times. Using the compiled information, what was the probability you drew a club?

Draw a card and replace it twenty more times. Using the compiled information, what was the probability you drew a club?

Do you notice anything of mathematical significance?

Using your last experimental probability, if you were to draw and replace a card 100 times, how many cards would you expect to be clubs? Would you actually draw that many clubs?

Homework #2

Name: ___________________________________Date: _________________

Please complete at least one problem from each section. Unless specified, write probabilities as fractions.

Section 1:

1. Toss a coin 10 times. Number of Heads _______________
   Number of Tails _______________
   What is the P(tails)?

2. Toss a coin 25 times. Number of Heads _________________________
   Number of Tails __________________________
   What is the P(tails)?

3. Toss a coin 50 times. Number of Heads _______________________________
   Number of Tails _________________________________
   What is the P(tails)?

Section 2:

4. You roll a number cube 100 times and get five 15 times. What is P(5)?

5. You spin a spinner 225 times and get red 30 times. What is P(red)?

6. You toss a coin 500 times and get a head 210 times. What is P(tail)?

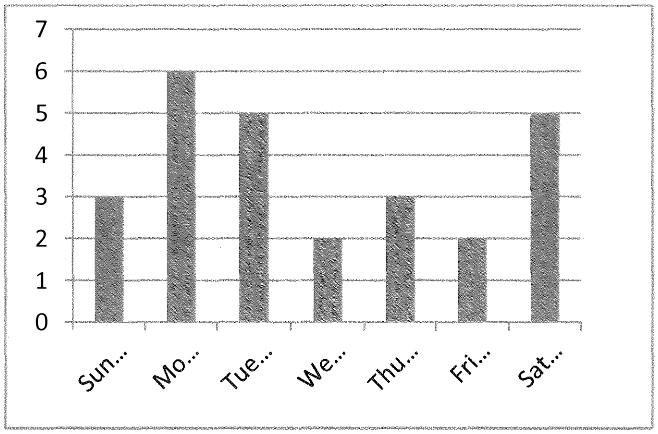
Section 3:

7. The probability that a male is colorblind is 8%. Suppose you interview 1000 males. About how many would you expect to be colorblind? Will you get exactly this number? Explain.
8. Knob Company estimates that on any day it makes x defective doorknobs. On Monday the total number of doorknobs is 252. Express the experimental probability of defective doorknobs on Monday in terms of x. If the experimental probability is $\frac{1}{42}$, what is x?
9. The experimental probability of winning for a team that has 3 wins and 2 losses is what? Using this probability, if they play a total of 15 games, how many will they win?

Section 4:

Use the data in the line plot. Find the experimental probability as a fraction in simplest form.

Students' Birthdays



10. P(Sunday)

11. P(weekday)

12. P(Wednesday)

Section 5:

Match the appropriate simulation with the scenario. Justify your answer.

a) Roll six number cubes 30 times.
b) Toss six coins 10 times
c) Choose a card from each of two groups of cards numbers 1 to 6. Repeat 40 times.

13. To win a game, you must guess two whole numbers from 1 to 6. What is the probability that you will guess both numbers?

14. The Golden Hen Egg Company packs eggs in groups of six per carton. The probability that an egg is cracked is 1/6. What is the probability that an egg carton will contain exactly two cracked eggs?

15. A student guesses the answers on six true/false quiz questions. What is the probability that the student will guess exactly two answers correctly?

Day 3: Sample Spaces LD

1. Understanding meaning

What do you think a sample space is?

Look it up! (pg 647)

2. Ways to organize sample space

Suppose you are going to flip a coin twice. What are all of the possible outcomes?

Suppose you are going to roll of standard die twice. What are all of the possible outcomes?

When these problems get more complicated, we need a systematic approach to display the sample space.

a. Make a Table:
   For flipping a coin your table would look like:

	H	T
H	H, H	H, T
T	T, H	T, T

   What would your table look like for the die problem?

For even more complex problems, you may use "table thinking" as a systematic way to list all of the options as opposed to actually making the table.

b. Tree Diagram

For the flipping a coin, your tree diagram would look like:

```
      H
   H /
  /  \ T
 <
  \  T – H
      \
       T
```

What would your tree diagram look like for the die problem?

3. How many items are in the sample space?

For the flipping of the coin two times there were 4 possibilities.

Think, a coin has 2 options and we flipped it 2 times and got 4.

For the rolling of the die, two times there were 36 possibilities.

Think, a die has 6 options and we flipped it 2 times and got 36.

How are we getting those numbers?

What if we flipped a coin and rolled a die, how many total possible outcomes would there be?

Did your above pattern hold? If not, how could you change it?

Check your pattern with me, if you have it right it is called the *counting principle.*

Use it to answer this question. At the deli there are five kinds of bread, six kinds of meat, and 2 kinds of spread. How many different sandwiches can be made with these options?

Day 3: Sample Spaces UD

4. Understanding meaning

What do you think a sample space is?

Look it up! (pg 647)

5. Ways to organize sample space

Suppose you are going to flip a coin twice. What are all of the possible outcomes?

Suppose you are going to roll of standard die twice. What are all of the possible outcomes?

When these problems get more complicated, we need a systematic approach to display the sample space.

c. Make a Table:
   For flipping a coin fill in the following table:

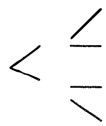
	H	T
H		
T		

What would your table look like for the die problem?

For even more complex problems, you may use "table thinking" as a systematic way to list all of the options as opposed to actually making the table.

d. Tree Diagram

For the flipping a coin, fill in the tree diagram:



What would your tree diagram look like for the die problem?

6. How many items are in the sample space?

For the flipping of the coin two times there were 4 possibilities.

For the rolling of the die, two times there were 36 possibilities.

How are we getting those numbers?

What if we flipped a coin and rolled a die, how many total possible outcomes would there be?

Did your above pattern hold? If not, how could you change it?

Check your pattern with me, if you have it right it is called the *counting principle.*

Use it to answer this question. At the deli there are five kinds of bread, six kinds of meat, and 2 kinds of spread. How many different sandwiches can be made with these options?

Homework #3

Name: ______________________________________Date: ___________________

Please complete at least one problem from each section. Unless specified, write probabilities as fractions.

Section 1:

Make a table to show the sample space for each situation and find the number of outcomes. Then find the probability.

1. You toss three coins. What is the probability of tossing two tails and one head?

2. You roll a standard number cube. What is the probability of rolling a number less than 4?

3. You toss a coin and spin a spinner. The spinner has four equal sections numbers from 1 to 4. What is the probability of tossing a tail and spinning a 4?

Section 2:

Make a tree diagram for each situation. Then find the probability.

4. A spinner is half red and half blue. What is the probability you spin it twice and get red both times?

5. You choose one letter at random from each of two sets of letters: A, B, C and W, X, Y, Z. What is the probability you get A and W?

6. You choose at ransom from the letters A, B, C, and D, and you roll a standard number cube. What is the probability you get A and 5?

Section 3:

Find the number of outcome for each situation.

7. Roll three number cubes.

8. Pick one of 7 boys and one of 12 girls.

9. Spin a spinner numbered 1 to 3, spinning four times.

Section 4:

Use the counting principle.

10. You are making a recipe with herbs and spices for a party. There are four types of herbs in your kitchen – basil, bay leaf, chives, and dill. You also have three types of seasoning – salt, pepper, and garlic powder. How many different recipes with one herb and one spice can you make?

11. A student takes classes in art, music, and history. There are four art teachers, three music teachers, and eight history teachers. How many courses taught by different teachers can the student take?

12. Suppose you have four jackets (white, blue, green, and tan) and four shirts in the same colors. How many different jacket – shirt outfits do you have?

Section 5:

13. At the cinema, there are three sizes of popcorn, four sizes of lemonade, and four sizes of fruit punch. How many orders are possible for popcorn and a beverage? List them.

14. Why is it sometimes helpful to use the counting principle instead of a sample space to find the outcomes of an event? When is it not helpful?

15. A traveler chooses a tour of Baltimore, Maryland, at random from buses D, E, and F. After the bus tour, she chooses a harbor tour at random from boats 1, 2, and 3. What is the probability she takes a tour with bus D and boat 2?

Day 4: Compound Events LD

1. Explore

Using the marbles in your supply basket, what is the probability of drawing a blue marble?

Using the marbles in your supply basket, what is the probability of drawing a red marble?

How do you think we would find the probability of drawing a blue marble, replacing it, and then drawing a red marble? (You are either going to add or multiply your two fractions. Which seems more practical?)

Check your hypothesis with me.

If you were correct you found the probability of a *compound event*!

2. Two types of Compound Events

a. **Independ**ent

What do you think?

Look it up! (pg 654)

What is the probability of drawing a green marble, replacing it, and then drawing a red marble?

What is the probability of spinning a 2 and then rolling a 2?

b. **Depend**ent

What do you think?

Look it up! (pg 655)

What is the probability of drawing a green marble, not replacing it, and then drawing a blue marble? (Think, "What is the probability of drawing a green marble, then, what is the probability of drawing a blue marble after the sample space changes?")

What is the probability of drawing a club, not replacing it, and then drawing a queen of hearts?

Day 4: Compound Events UD

3. Explore

Using the marbles in your supply basket, what is the probability of drawing a blue marble?

Using the marbles in your supply basket, what is the probability of drawing a red marble?

How do you think we would find the probability of drawing a blue marble, replacing it, and then drawing a red marble?

Check your hypothesis with me.

If you were correct you found the probability of a *compound event*!

4. Two types of Compound Events

c. Independent

What is it?

Look it up! (pg 654)

What is the probability of drawing a green marble, replacing it, and then drawing a red marble?

What is the probability of spinning a 2 and then rolling a 2?

d. Dependent

What is it?

Look it up! (pg 655)

What is the probability of drawing a green marble, not replacing it, and then drawing a blue marble?

What is the probability of drawing a club, not replacing it, and then drawing a queen of hearts?

What is the probability of rolling an even number on a standard die, drawing a green marble, not replacing it, and then drawing a red marble?

Homework #4

Name: ______________________________________Date: ___________________

Please complete at least one problem from each section. Unless specified, write probabilities as fractions.

Section 1:

You roll a standard number cube twice. Find the probability

1. P(1, then 2)

2. P(3, then even)

3. P(greater than 2, then odd)

Section 2:

An arrangement of 8 students is shown. The names of all the students are in a basket. The teacher draws one name and replaces it. Then the teacher draws a second name. Find each probability.

Row	Student			
A	1	2	3	4
B	5	6	7	8

4. P(student 1, then student 8)

5. P(a student in row A, then a student in row B)

6. P(a student in row A, then student 6, 7, or 8)

Section 3:

A bag contains 3 blue marbles, 4 red marbles, and 2 white marbles. Three times you draw a marble and return it. Find the probability.

7. P(red, then white, then blue)
8. P(red, then blue, then, blue)
9. P(all blue)

Section 4:

Are the two events independent or dependent?

10. You toss a nickel and then you toss a dime.

11. You draw a card and then you draw another.

12. You grab a sock from the dyer and then grab another sock from the dryer.

Section 5:

13. Events with no outcomes in common are disjoint events. To find the probability of disjoint events add the probabilities of the individual events. Suppose you select a number from 21 to 30 at random. What is the probability of selecting a number that is even or prime?

14. You have two spinners with colors on them. The probability of spinning "green" on both spinners is $\frac{5}{21}$. The probability of spinning "green" on the first spinner alone is $\frac{1}{3}$. What is the probability of spinning "green" on the second spinner alone?

15. Five girls and seven boys want to be the two broadcasters for a school variety show. To be fair, a teacher puts the names of the student in a hat and draws two. Find P(girl, then boy).

Day 5: Permutations LD

1. Explore

Figure out how many ways your group can line up two people to go get a drink.

Figure out how many ways your group can line up three people to go get a drink.

> Mathematically, how could you use the number of people in your group and the size of the groups you are forming to get the number of arrangements? For example, could you multiply the size of the groups by the number of people total? Do we multiply the number of people but change that number each time someone has joined the group?

Try to use your discovery to find how many ways your group can line up four people to go get a drink. Does your answer make sense?

Come check your hypothesis with me.

When you do ______________________ it is called the ______________________ of a number.

2. Understanding meaning.

Based on what you were just doing what do you think a permutation is?

Look it up! (pg 660)

3. Examples (Use your supply basket if necessary)

How many ways can you hand three coins to a cashier one at a time from a penny, nickel, dime, and quarter?

Suppose your class is having a spelling bee. How many different outcomes are there for rank?

Day 5: Permutations UD

1. Explore

Figure out how many ways your group can line up two people to go get a drink.

Figure out how many ways your group can line up three people to go get a drink.

Mathematically, how could you get the number of groups?

Try to use your discovery to find how many ways your group can line up four people to go get a drink. Does your answer make sense?

Come check your hypothesis with me.

When you do ______________________ it is called the _____________________ of a number.

2. Understanding meaning.

Based on what you were just doing what is a permutation?

Look it up! (pg 660)

3. Examples (Use your supply basket if necessary)

How many ways can you hand three coins to a cashier one at a time from a penny, nickel, dime, and quarter?

Suppose your class is having a spelling bee. How many different outcomes are there for rank?

Homework #5

Name: ____________________________________Date: __________________

Please complete at least two problems from each section.

Section 1:

Write the number of permutations in factorial form. Then, simplify the number of permutations for the group of letters.

1. W, O, R, L, D

2. T, O, Y

3. L, U, N, C, H, E, S,

Section 2:

Find the number of two-letter permutations of the letters.

4. M, A, P, L, E

5. A, B, C, D, E, F, G, H

6. X, Y, Z

Section 3:

7. Suppose you plan to shop, call a friend, study, and exercise on a weekend day. How many arrangements of activities can you plan?

8. Suppose you scramble the letters P, A, N. Make an organized list of the sample space. How many of the groups form real words and what are they?

9. The password to a school computer has 3 letters of the alphabet. You don't have the password. Supposed you try different arrangements. If you are as unlucky as possible, how many will you have to try before you have access to the computer?

Day 6: Combinations LD

1. Explore

Figure out how many ways your group can form a two person committee to plan chapels.

Figure out how many ways your group can form a three person committee to plan chapels.

Mathematically, how could you use the number of people in your group and the size of the groups you are forming to get the number of committees? Yesterday, we multiplied the size of the group by one less than the size of the group for each number in the arrangement. Today we want to take into account overlap, so what could we divide by to get the number of committees?

Try to use your discovery to find how many ways your group can form a four person committee to plan chapels. Does your answer make sense?

Come check your hypothesis with me.

2. Understanding meaning.

Based on what you were just doing what do you think a combination is?

Look it up! (pg 664)

3. Examples (Use your supply basket if necessary)

How many ways can you hand three coins at once to a cashier from a penny, nickel, dime, and quarter?

Suppose your class is going on a field trip. One group of four will get to ride in my car. How many different options are there for students riding with me?

4. Important to Remember!!

For permutations order _____________. For combinations order_______________.

Day 6: Combinations UD

1. Explore

Figure out how many ways your group can form a two person committee to plan chapels.

Figure out how many ways your group can form a three person committee to plan chapels.

Mathematically, how could you get the number of committees?

Try to use your discovery to find how many ways your group can form a four person committee to plan chapels. Does your answer make sense?

Come check your hypothesis with me.

2. Understanding meaning.

Based on what you were just doing is a combination?

Look it up! (pg 664)

3. Examples (Use your supply basket if necessary)

How many ways can you hand three coins at once to a cashier from a penny, nickel, dime, and quarter?

Suppose your class is going on a field trip. One group of four will get to ride in my car. How many different options are there for students riding with me?

4. Important to Remember!!

For permutations _______________. For combinations _______________________.

Homework #6

Name: ______________________________________Date: ___________________

Please complete at least one problem from each section. Unless specified, write probabilities as fractions.

Section 1:

Find the number of combinations.

1. Choose two people from three.

2. Choose two people from six.

3. Choose three people from five.

Section 2:

Use the letters B, E, O, P, R, W. Make a list of all of the combinations.

4. 3 consonants

5. Any 5 letters

6. 2 vowels

Section 3:

7. You have 5 different CDs to play. Your CD player can hold 3 CDs. How many different combinations of 3 CDs can you select?

8. A club of 50 people wants to select 4 members to represent them. How many different combinations of 4 people are possible?

9. You want to mix some paint colors, choosing from blue, green, yellow, and red. How many combinations of two paints are possible? Suppose you choose two colors at random. Find P(blue and green).

Section 4:

Determine if the situation involves a combination or a permutation. Then answer the question.

10. You select three books from a bookshelf that holds eight books. How many different sets of books can you choose?

11. You have six pizza toppings to use for a pizza. How many different three – topping pizzas can you make?

12. Four students stand beside each other for a photograph. How many different orders are possible?

Section 5:

13. To open a combination lock, you must dial the numbers in the right order. Explain why "permutation lock" might be more appropriate than "combination lock" as a name for this lock.

14. Explain the difference between a permutation and a combination. Give an example of each.

15. Write your own combination problem. How would you reword it to make it a permutation problem? Solve your problem both ways.

Appendix B

Interview Questions LD: Wander, Cymbalta Overdose, Rori

1. *What was your opinion of class during the research period?*

Wander: The probability thing for school has made it harder. I feel like I haven't learned anything in this unit because we had to teach ourselves how to do it and I didn't have any idea how to do it.

What about when I would go over it the next day?

That helped but then I felt like class time was confusing and a waste.

Rori: The group idea, even though I get why we did it, didn't really help me. I think it hurt me more.

Why?

I never understood what we were doing until after the fact and even then I'm not sure if I did.

Did you enjoy the class time?

Cymbalta Overdose: It was better than just taking notes but still confusing.

2. *What was the opinion of your class group?*

Rori: People in the groups would want to play around and not do the work or there were others who would want to do all the work and have us copy so we could get it done faster, others wouldn't want to do anything at all and just sit there and complain.

Wander: I agree, the groups were weird and not always productive.

Cymbalta Overdose: My groups were ok; just we weren't the best to be learning from each other.

a. *Would you have chosen to be in the group you were assigned or the other group? Why?*

Wander: I would have chosen to have some of the smarter people with me so that they could have helped me.

Cymbalta Overdose: Agreed, I wish I could have had *Mathisfun* in my group or *Philipes Bananas*. *Rori*, at least you got to have *Amazing Person* in your group.

Rori: Yeah, but he was the one always rushing to get done.

3. a. *What was your opinion of math before the research period?*

Wander: Sometimes I do well and I like math pretty well but other times not so much. Before this started, I liked it better than at the beginning of the year and I wanted to be able to learn and come to math class when other times I didn't.

Cymbalta Overdose: I just struggle in math in general. It's a really hard effort for me that I don't enjoy putting in because when I'm not in math I can just relax.

Rori: I didn't think I was that good in math but it's ok. I'm not good at staying organized which doesn't help.

b. *What is your opinion of math now?*

Rori: Not much has changed, it's ok but I don't love it.

Wander: Now I really did not like it and I am not so eager to learn math any more.

Why?

I just feel like I'm really bad at math and this unit showed that to me. I couldn't figure anything out.

Cymbalta Overdose: I still think math is something I cannot stand and I just don't understand it.

4. What would you have changed about class while learning probability?

Cymbalta Overdose: I just think I do better one on one and have extra help. I should have come and gotten that during study hall.

Wander: I would have just learned it the normal way.

Rori: I would have had us work in other groups. Ones that may have helped us learn more.

Interview Questions UD: Nobles, Philipes Bananas, Dolphins1254

5. *What was your opinion of class during the research period?*

Nobles: I liked the groups better than you talking.

Why?

Less boring.

Dolphins1254: It definitely wasn't boring but depending on which group I was in, I enjoyed it less or more.

6. *What was the opinion of your class group?*

Philipes Bananas: I didn't like my first group but I did like my second one.

Why?

The people in the first group didn't really want to work but in the second group we got a lot done and learned what we should.

Dolphins1254: I was the opposite, I liked the first group and then you switched us.

Why?

Because I was with friends and then not.

Nobles: I didn't really care which group I was in, I didn't love either but I didn't hate them.

a. *Would you have chosen to be in the group you were assigned or the other group? Why?*

Nobles: I don't know.

Dolphins1254: No, I wanted to be with friends the whole time. We should have gotten to pick our groups.

Philipe's Bananas: That wasn't a part of her study! I would have chosen my second group. I work well with those students and we all like math.

7. a. *What was your opinion of math before the research period?*

Philipes Bananas: Math is easy for me and I don't have to study but I still get grades 95 or above on most tests. I get really excited when we learn something new in math class.

Nobles: Math is ok. I'm good at it but I don't really like it. I like Tuesdays when we don't have math class.

Dolphins1254: I agree. Math is ok.

b. *What is your opinion of math now?*

Nobles: The same.

Dolphins1254: I get everything but probability. I don't think I learned well that way but I would still say I'm a good math student.

Philipe's Bananas: The same, math is still pretty easy for me and I still usually don't have to study for tests in math.

c. *What would you have changed about class while learning probability?*

Nobles: I wish we still had a smartboard. I wish we could put more money towards math, then it would be fun.

Why would it be more fun?

It would be more interactive and interesting.

But wasn't the group work interactive?

Yeah, but it's not the same.

Philipe's Bananas: I think a little more explanation from you would have been helpful to end class but it was good that you went over the homework the next day. I liked our independence.

Dolphins1254: Let us pick our own groups!! Also, I don't think we need a smartboard; it's not that helpful or interesting.

Appendix C

Marvel

Please mark the box that best matches your opinion.
(SD = Strongly Disagree, D = Disagree, A = Agree, SA = Strongly Agree)

Statement	SD	D	A	SA
1. Mathematics is something which I enjoy very much.		X		
2. I do not do very well in mathematics.			X	
3. Doing mathematics problems is fun.	X			
4. I would like to do some outside reading in mathematics.		X		
5. Mathematics is easy for me.	X			
6. When I hear the word mathematics, I have a feeling of dislike.		X		
7. I would like to spend less time in school doing mathematics.		X		
8. Sometimes I read ahead in our mathematics book.	X			
9. I usually understand what we are talking about in mathematics class.		X		
10. No matter how hard I try, I cannot understand mathematics.		X		
11. I would like a job which does not use any mathematics.			X	
12. I enjoy talking to other people about mathematics.		X		
13. I am good at doing mathematics problems.		X		
14. Sometimes I do more mathematics problems than are given in class.	X			
15. I remember most of the things I learn in mathematics.		X		
16. I have a good feeling towards mathematics.			X	
17. I have a real desire to learn mathematics.				X

Other Comments: Math is essential to life, though I find it very annoying, but I still want to fully understand it.

Math is fun

Please mark the box that best matches your opinion.
(SD = Strongly Disagree, D = Disagree, A = Agree, SA = Strongly Agree)

Statement	SD	D	A	SA
1. Mathematics is something which I enjoy very much.				✓
2. I do not do very well in mathematics.	✓			
3. Doing mathematics problems is fun.				✓
4. I would like to do some outside reading in mathematics.		✓		
5. Mathematics is easy for me.				✓
6. When I hear the word mathematics, I have a feeling of dislike.	✓			
7. I would like to spend less time in school doing mathematics.	✓			
8. Sometimes I read ahead in our mathematics book.			✓	
9. I usually understand what we are talking about in mathematics class.				✓
10. No matter how hard I try, I cannot understand mathematics.	✓			
11. I would like a job which does not use any mathematics.	✓			
12. I enjoy talking to other people about mathematics.				✓
13. I am good at doing mathematics problems.				✓
14. Sometimes I do more mathematics problems than are given in class.		✓		
15. I remember most of the things I learn in mathematics.				✓
16. I have a good feeling towards mathematics.				✓
17. I have a real desire to learn mathematics.				✓

Other Comments: Math is the best! I want to be a math teacher when I am older.

Eggroll

Please mark the box that best matches your opinion.
(SD = Strongly Disagree, D = Disagree, A = Agree, SA = Strongly Agree)

Statement	SD	D	A	SA
1. Mathematics is something which I enjoy very much.			✓	
2. I do not do very well in mathematics.			✓	
3. Doing mathematics problems is fun.				✓
4. I would like to do some outside reading in mathematics.		✓		
5. Mathematics is easy for me.				✓
6. When I hear the word mathematics, I have a feeling of dislike.	✓			
7. I would like to spend less time in school doing mathematics.		✓		
8. Sometimes I read ahead in our mathematics book.		✓		
9. I usually understand what we are talking about in mathematics class.				✓
10. No matter how hard I try, I cannot understand mathematics.	✓			
11. I would like a job which does not use any mathematics.	✓			
12. I enjoy talking to other people about mathematics.				✓
13. I am good at doing mathematics problems.				✓
14. Sometimes I do more mathematics problems than are given in class.			✓	
15. I remember most of the things I learn in mathematics.				✓
16. I have a good feeling towards mathematics.			✓	
17. I have a real desire to learn mathematics.				✓

Other Comments: I ♡ Math

Appendix D

0/17=0% I DON'T KNOW THIS STUFF

Probability Test

Name: Crimenal Date:

Answer the following problems, showing all necessary calculations.
Unless specified, write probabilities as fractions.

1. Six balls numbered from 1 to 6 are placed in an urn. One ball is selected at random. Find the probability that it is NOT number 5.

NA ?

2. A single fair number cube is tossed. Find the probability of obtaining a number other than 9.

NA ?

3. There are 10 crayons in a box. Two are orange. A crayon is selected at random. Find the probability that it is orange as a fraction, decimal, and percent.

NA ?

4. The table shows the drink preferences of 50 shoppers at the mall. What is the probability that one shopper, selected at random from the 50 surveyed, preferred either Drink A or Drink C?

NA ?

Drink Survey

Drink	Number of Shoppers
A	12
B	7
C	14
D	6
E	11

5. From a barrel of colored marbles, you randomly select 7 blue, 5 yellow, 8 red, 4 green, and 6 purple marbles. Find the experimental probability of selecting a marble that is NOT purple.

NA ?

6. A Lights-A-Lot quality inspector examines a sample of 25 strings of lights and finds that 3 are defective. What is the best prediction of the number of defective strings of lights in a delivery of 150 strings of lights?

NA ?

7. A bag contains 2 purple marbles and 4 green marbles. One marble is drawn and not replaced. Then a second marble is drawn. What is the probability that the first marble is green and the second one is purple?

NA ?

8. A bag contains five blue marbles and one white marble. The marbles are randomly selected one at a time. What are the odds of picking the white marble on the first selection?

$\frac{50}{50}$

9. A multiple-choice test has 5 questions, each with 4 possible answers. Find the probability of guessing the correct answers to all of the questions.

? $\frac{1}{\text{in a million}}$ - idk!

10. A car manufacturing makes three cars: a hatchback, a coupe, and a sedan. These cars can come in three different colors: silver, grey, and black. Make a tree diagram to show the sample space for the different selections. What is the probability of randomly selecting a yellow coupe?

NA ?

11. Use the counting principle to determine how many different meals can be made from 3 beverages, 4 sandwiches, 3 sides, and 2 desserts.

NA ?

12. How many different ways can you arrange five people shoulder-to-shoulder in a line?

1,000,000,000. idk im dumb.

13. There are 14 students participating in a spelling bee. How many ways can the students who go first, second, third, fourth and fifth be chosen?

alphabetically, randomly, gender, age, idk!

14. Four cards are drawn in succession, without replacement, from a standard deck of 52 cards. How many sets of four cards are possible?

NA ?

15. A committee is to have three members. There are seven men and four women available to serve on the committee. How many different committees can be formed?

NA ?

Appendix E

Code	HW 1 # Completed	HW 1 # Correct
Zamboni 37	1.2.3.4.5.6.7.8.9.10.11.12.13.15	1.2.3.4.6.{10.11.12}(1/2).13.15
CLA Butterfly	1.2.3.4.5.6.7.8.9.10.11.15	1(1/2).2.4.5.6.10.11(1/2).15
Marvel	2.4.7.10.15	4.10(1/2).15
19-12	1.6.7.12.14	6.12
Jerico	3.4.5.6.7.8.9.12.15	3.4.5.6.12(1/2).15
Nobles	1.2.3.4.5.6.7.8.9.10.15	1.2.3.4.5.6.10.15
Criminal	1.6.7.10.14	
CJsports3	3.5.7.10.15	3.5.15
Philipe's Bananas	3.6.7.8.9.10.13	6.10(1/2)
Cymbalta Overdose	2.4.7.10.15	2.4.10.15
Rori	1.4.9.11.14	4.9.11(1/2)
Wander	1.3.4.5.9.10.15	3(1/2).4.5.15
Dolphins1254	1.2.3.4.5.6.7.8.9.10.11.13.14.15	2.3.4
Achmed	1.2.4.7.8.9.10.11.12.13	1.2.4.13
Amazing Person	1.4.7.12.15	4.7.15
Blue	2.4.5.9.11. 13	2.4.13
Eggroll	2.6.8.10.15	8.10(1/2).15
Mathisfun	1.2.3.4.5.6.7.8.9.10.11.12.15	1.2.3.4.5.6.10.11.12.15

Code	HW 2 # Completed	HW 2 # Correct
Zamboni 37	1.4.5.6.7.8.9.10.11.12.13.14.15	1.4.5.7.10.11.12.13.14.15
CLA Butterfly	1.2.4.5.6.9.10.13	1.2.4.5.10
Marvel	1.2.3.4.9.10.15	1.2.3.4.9.10.15
19-12	1.4.7.10.13	1.4.7.10
Jerico	1.6.9.10.13	6.9.10.13
Nobles	1.4.7.10.14	1.4.7.14
Criminal	1.4.7.10.15	7
CJsports3	1.4.9.12.13	1.9.12.13
Philipe's Bananas	1.4.7.12.13.14.15	1.4.7(1/2).12.13(1/2).14(1/2).15
Cymbalta Overdose	1.2.4.9.10.15	1.2.4(1/2).9.10(1/2).15
Rori	1.5.7.10.13	1.7
Wander	1.2.4.8.10.12.15	4.8.15
Dolphins1254	1.2.4.6.7.10.12.13.14.15	1.2(1/2).7.10.12.13.
Achmed	1.2.3.4.7.10.13	1.2.4.7
Amazing Person	1.4.7.10.13	4.10.13
Blue	1.4.6.7.10.15	1.4.6.7(1/2).10
Eggroll	1.4.7.10.15	4.7(1/2).10.15
Mathisfun	1.4.5.6.7.10.11.12.13	4.5.7.10.11.12

Code	HW 3 # Completed	HW 3 # Correct
Zamboni 37	1.2.4.9	
CLA Butterfly	1.2.4.7.10.13	4(1/2).10
Marvel	2.4.7.12.13	4(1/2).7(1/2).12.13(1/2)
19-12	1.5.8.12.13	
Jerico	1 to 15	1 to 15
Nobles	2.4.7.12.14	2(1/2).12.14
Criminal	1.4.7.12.15	12
CJsports3	2.4.9.10.13	2(1/2)
Philipe's Bananas	2.6.9.12.15	6(1/2).12
Cymbalta Overdose	2.6.7.12.15	6(1/2).12
Rori	-	-
Wander	1.5.9.12.13	12.13
Dolphins1254	1.2.3.4.9.11.12.15	2(1/2)
Achmed	1.4.9.12.13	12
Amazing Person	2.4.7.12.13	2(1/2).4(1/2).12
Blue	1.4.7.10.15	4.10.15
Eggroll	1.4.6.9.12.13	1.4.6(1/2).12
Mathisfun	1.4.5.7.10.12.13.15	4.5(1/2).10.12.15

Code	HW 4 # Completed	HW 4 # Correct
Zamboni 37	1.2.3.4.5.6.7.10.15	1.2.3.4.5.6.7.10
CLA Butterfly	1.4.7.10.13	1.4.10
Marvel	1.4.9.11.15	1.4.9.11
19-12	2.4.7.10.15	4.1
Jerico	1 to 15	1 to 15
Nobles	1.4.7.10.14	10
Criminal	1.4.7.9.1 1.15	11
CJsports3	1.5.9.12.13	1.5.9
Philipe's Bananas	2.4.5.7.9.10.11.12.14	4.5.7(1/2).10.11.12
Cymbalta Overdose	1.4.9.10.15	10
Rori	1.4.7.11.13	11
Wander	1.4.9.10.11.12	1.4.10.1 1.12
Dolphins1254	1.4.7.10.12.15	1.4
Achmed	1.4.7.12.14	4(1.2).12.14
Amazing Person	1.4.7.10.15	1.4.7.10
Blue	1.4.9.10.11.12.13	1.4
Eggroll	1.4.9.12.15	4.9
Mathisfun	1.2.3.4.5.6.7.10.11.12.14.15	1.2.3.4.5.6.10.11.12.14

Code	HW 5 # Completed	HW 5 # Correct
Zamboni 37	8	8
CLA Butterfly	1.4.7	1.4
Marvel	2.4.8	2.6.8
19-12	1.2.3.4.5.6.7.9	7
Jerico	1 to 9	1 to 9
Nobles	2.6.8	2.6.8(1/2)
Criminal	1.2.4.6.7.8	8(1/2)
CJsports3	1.2.4.6.7.8	1.2.4.6.8
Philipe's Bananas	1.2.3.4.5.6.7.9	1.2.3.4.5.6.7.9
Cymbalta Overdose	2.6.8	2.6.8
Rori	1.2.4.6.7.8	1.2.6.8(1/2)
Wander	2.4.7.9	
Dolphins1254	1.2.4.6.8.9	1.2.4.6.8(1/2)
Achmed	1.2.4.5.6.8.9	1.2.6
Amazing Person	1.2.4.6.7.8	1.2.4.6.7.8
Blue	1.2.4.6.7.8	1.2.4.6.7
Eggroll	1.2.4.6.7.8	1.2.4.6.7.8
Mathisfun	1.2.3.4.5.6.7.9	1.2.3.5.6

Code	**HW 6 # Completed**	**HW 6 # Correct**
Zamboni 37	1.4.7.10	
CLA Butterfly	1.4.7.10.14	1.4.7(1/2).14
Marvel	1.6.9.12.14	1.6.9(1/2).14
19-12	1.6.7.10.13	6.13
Jerico	1 to 14	1.2.3.4(1/2).5.6.7.8(1/2).9.10(1/2).11(1.2).13.14
Nobles	1.5.7.10.13	1.5.13
Criminal	-	-
CJsports3	1.6.7.11.14	6.14
Philipe's Bananas	1.2.5.7.10.13	1.5.13
Cymbalta Overdose	2.5.8.10.13	2.13
Rori	1.5.7.10.	1.5
Wander	1.4.6.8.11.14	11(1/2).14
Dolphins1254	1.6.7.10.13	1.10(1/2).13
Achmed	1.6.7.10.14	1.10(1/2)
Amazing Person	1.6.7.12.14	1.12(1/2).14
Blue	1.6.8.11.13	1
Eggroll	1.4.5.7.12.14	1.5.12.14
Mathisfun	1.2.3.4.6.7.8.12.13	1.4.6.12.13

Appendix F

Homework #2

Name: 19-12 Date: ____________

Please complete at least one problem from each section. Unless specified, write probabilities as fractions.

Section 1:

1. Toss a coin 10 times. Number of Heads ~~||||~~ || $\frac{2}{10} = \frac{1}{5}$
   Number of Tails ||
   What is the P(tails)?

2. Toss a coin 25 times. Number of Heads ____________
   Number of Tails ____________
   What is the P(tails)?

3. Toss a coin 50 times. Number of Heads ____________
   Number of Tails ____________
   What is the P(tails)?

Section 2:

4. You roll a number cube 100 times and get five 15 times. What is P(5)? $\frac{15}{100}$

5. You spin a spinner 225 times and get red 30 times. What is P(red)?

6. You toss a coin 500 times and get a head 210 times. What is P(tail)?

Section 3:

7. The probability that a male is colorblind is 8%. Suppose you interview 1000 males. About how many would you expect to be colorblind? Will you get exactly this number? Explain.
   80 No because there

8. Knob Company estimates that on any day it makes x defective doorknobs. On Monday the total number of doorknobs is 252. Express the experimental probability of defective doorknobs on Monday in terms of x. If the experimental probability is $\frac{1}{42}$, what is x?

9. The experimental probability of winning for a team that has 3 wins and 2 losses is what? Using this probability, if they play a total of 15 games, how many will they win?

3/6

Homework #6

Name: ______________________ Date: ______________

Please complete at least one problem from each section. Unless specified, write probabilities as fractions.

Section 1:

Find the number of combinations.

1. Choose two people from three.

   3

2. Choose two people from six.

   720

3. Choose three people from five.

Section 2:

Use the letters B, E, O, P, R, W. Make a list of all of the combinations.

4. 3 consonants

5. Any 5 letters

   6

6. 2 vowels

Section 3:

7. You have 5 different CDs to play. Your CD player can hold 3 CDs. How many different combinations of 3 CDs can you select?

   20

8. A club of 50 people wants to select 4 members to represent them. How many different combinations of 4 people are possible?

9. You want to mix some paint colors, choosing from blue, green, yellow, and red. How many combinations of two paints are possible? Suppose you choose two colors at random. Find P(blue and green).

Appendix G

Section 3:

Find the number of outcome for each situation.

7. Roll three number cubes.

60

8. Pick one of 7 boys and one of 12 girls.

9. Spin a spinner numbered 1 to 3, spinning four times.

Section 4:

Use the counting principle.

10. You are making a recipe with herbs and spices for a party. There are four types of herbs in your kitchen – basil, bay leaf, chives, and dill. You also have three types of seasoning – salt, pepper, and garlic powder. How many different recipes with one herb and one spice can you make? 12

11. A student takes classes in art, music, and history. There are four art teachers, three music teachers, and eight history teachers. How many courses taught by different teachers can the student take?

12. Suppose you have four jackets (white, blue, green, and tan) and four shirts in the same colors. How many different jacket – shirt outfits do you have?

Section 5:

13. At the cinema, there are three sizes of popcorn, four sizes of lemonade, and four sizes of fruit punch. How many orders are possible for popcorn and a beverage? List them.

14. Why is it sometimes helpful to use the counting principle instead of a sample space to find the outcomes of an event? When is it not helpful?

15. A traveler chooses a tour of Baltimore, Maryland, at random from buses D, E, and F. After the bus tour, she chooses a harbor tour at random from boats 1, 2, and 3. What is the probability she takes a tour with bus D and boat 2?

$\frac{1}{9}$

Homework #4

Name: ______________________ Date: ______________

Please complete at least one problem from each section. Unless specified, write probabilities as fractions.

Section 1:

You roll a standard number cube twice. Find the probability

1. P(1, then 2)
   $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$
2. P(3, then even)
3. P(greater than 2, then odd)

Section 2:

An arrangement of 8 students is shown. The names of all the students are in a basket. The teacher draws one name and replaces it. Then the teacher draws a second name. Find each probability.

Row	Student			
A	1	2	3	4
B	5	6	7	8

4. P(student 1, then student 8)
5. P(a student in row A, then a student in row B)
6. P(a student in row A, then student 6, 7, or 8)

Section 3:

A bag contains 3 blue marbles, 4 red marbles, and 2 white marbles. Three times you draw a marble and return it. Find the probability.

7. P(red. then white, then blue)
8. P(red. then blue, then, blue)
9. P(all blue)

Section 4:

Determine if the situation involves a combination or a permutation. Then answer the question.

10. You select three books from a bookshelf that holds eight books. How many different sets of books can you choose?

11. You have six pizza toppings to use for a pizza. How many different three – topping pizzas can you make?

12. Four students stand beside each other for a photograph. How many different orders are possible?

Section 5:

13. To open a combination lock, you must dial the numbers in the right order. Explain why "permutation lock" might be more appropriate than "combination lock" as a name for this lock.

14. Explain the difference between a permutation and a combination. Give an example of each.

15. Write your own combination problem. How would you reword it to make it a permutation problem? Solve your problem both ways.

Vita

Mindy Marlene Swancott was born in New Hartford, NY on September 6, 1987. She attended Houghton College from 2005 to 2009 and received a Bachelor of Arts in Mathematics and in Adolescence Education in 2009. She began work toward a Master of Science in Education and Human Development, Adolescence Mathematics, at the State University of New York College at Brockport in the Fall of 2009.